



Estimating Elasticities for the Residential Demand of Electricity in Brazil Using Cointegration Models

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ABSTRACT

Estimates of demand elasticities are crucial for the development of a range of energy policy studies, especially in countries of continental dimensions like Brazil. The paper explores the class of cointegration models as a provider of competing methods to estimate income, electricity price, and equipment price elasticities for the residential demand of electricity in Brazil from 1974 to 2016. We compare two cointegration methods: the well-known Johansen's (1988) approach and the autoregressive distributed lag model of Pesaran et al. (2001). The use of the latter is novel in the electricity demand literature regarding Brazilian studies. The results show that the two models produce similar elasticities and indicate that the residential demand is stable. The long-run elasticities are much larger than the short-run elasticities. In the long run, we find that income is the main determinant of demand, while variations in electricity price and electric equipment price show modest effect on demand.

Keywords: Residential Electricity Demand, Demand Elasticities, Cointegration

JEL Classifications: Q41, Q43, C22

1. INTRODUCTION

Understanding demand causation in electricity markets is vital for producers and policy makers. Forecasts are used to determine reserve margins and elasticity estimates are necessary to compute demand changes in response to income and price variations. These are key information in electricity markets because electricity cannot be stored in substantial volumes, generation capacity is restricted, and retail markets use to be heavily regulated. Consequently, a vast literature has developed addressing the estimation of electricity demand functions.

A great concern of electricity markets' players is to discover essential features of electricity demand, in particular whether it is stable or not. A stable demand function underlies the view that a theoretical relationship between electricity demand and causal variables exists, so that policy proposals can be designed to meet current and future levels of electricity demand (Dergiades and Tsoufidis, 2008). If electricity demand is unstable, policy makers

can neither design meaningful energy policies nor build reliable forecasts of future demand for electricity.

Since the energy crisis of 2001, Brazilian regulatory authorities have tightened the rules over the distribution system, imposing fines and increasing costs for distributors that did not guarantee their power supply. Electricity demand forecasts are largely used in power system's planning, energy trading, and tariff (price) regulation. Electricity distribution prices are regulated by a price cap scheme in which at each period of 4 years the Brazilian Electricity Regulatory Agency (ANEEL) conducts periodic tariff revisions to promote gains in efficiency to the electricity distribution industry (Pessanha and Leon, 2015).

In order to help regulators properly make evaluations regarding the effects of a tax or a subsidy, ANEEL (2008) recommends that electricity distributors use multivariate time series econometric models to estimate the relationship between electricity demand and its determinants. ANEEL explicitly recommends the Johansen's

(1988) cointegration methodology, which is based on the vector error correction model (ECM), and the autoregressive distributed lag (ARDL) approach to cointegration proposed by Pesaran et al. (2001). Such methods can deal with the spurious regressions problem, which arise when at least two variables in the data generation model are non-stationary. The goal of this study was to compare the econometric models recommended by ANEEL with regard to their capacity to provide robust estimates mainly of income and price elasticities of electricity demand in Brazil, in particular for the residential sector. We used annual data from 1974 to 2016 to make the empirical analysis. To the best of our knowledge, our application of the ARDL approach is original, as we have found no similar study employing this method to analyze the residential demand of electricity in Brazil.

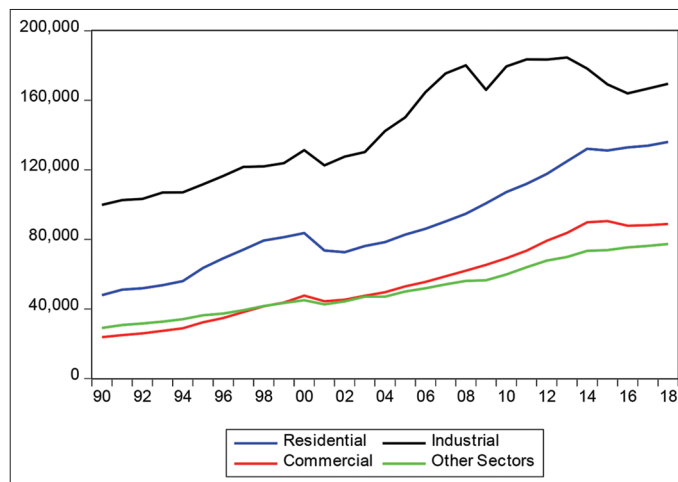
In addition to this introduction, this paper is structured as follows. Section 2 presents an overview of the residential consumption of electricity in Brazil. Section 3 makes a review of the literature about the estimation of models and elasticities for the residential demand of electricity. Section 4 explains the methodology used, say, the empirical model specification, the two econometric methodologies and the sources of data. Section 5 discusses the empirical results and Section 6 presents some conclusions.

2. ELECTRICITY INDUSTRY IN BRAZIL

In the last three decades, Brazil has shown a rapid increase of electricity consumption, with an average growth about 3% per year between 2003 and 2019. According to the 2020 *Statistical Yearbook of Electricity*¹ (EPE, 2020), among the ten largest consumers of electricity in the world, Brazil is behind only to India and China in terms of growth in energy consumption and, in 2017, surpassed Canada in absolute terms of TWh. Part of this performance was due to investments made in the expansion of the electricity transmission network throughout the country in the early 2000s, which allowed the interconnection of various regional electrical systems which were previously isolated. This has enabled spatial economic integration among the regional electricity markets, providing more competitive prices for consumers in addition to quality and security of power supply. Since 2003, the growth of electricity consumption in Brazil has been a common feature to all segments almost every year. An exception was the industrial sector, which was struck by the financial crisis of 2008 and the political crisis in 2014 (Figure 1). The residential sector has displayed the highest growth among consumption branches, increasing from 76 TWh in 2003 to 136 TWh in 2018. Thus, it is extremely important to study this sector not only for distributors to anticipate future demands, but also for planning the expansion of the electricity system.

Brazil underwent a severe electricity crisis in 2001. The lack of planning led to insufficient investments in the expansion of power generation, making the power system to approach a collapse when a sudden drought provoked a sharp drop of water reservoirs levels. The federal government responded a rationing scheme to reduce the country's electricity consumption. For the residential

Figure 1: Electricity consumption by major groups in Brazil (1990–2019)



Source: The electricity consumption is measured in TWh

sector, a specific rule was imposed requesting each household to restrain its electricity consumption by 20%. The rationing scheme was removed in February 2002, but caused a sharp break in the overall trend of the residential demand for electricity as we see in Figure 1. Hydroelectric power plants represent almost 65% of electricity generation in Brazil² (EPE, 2020), which makes the use of good elasticity estimates by electrical agents even more important in periods of low rainfall. Also in the 2000s, two other important public policies targeting the residential sector were implemented: the programs “Luz para Todos” (Light for All in English) and Procel (National Electrical Energy Conservation Program in English). Both programs have been sponsored by state agencies which received funds over several years to carry out their goals (MME, 2011). The “Luz para Todos” program’s goal was to provide universal access to electricity in Brazil, as a response to the large number of low-income Brazilian families who had no access to electricity in their residences. In 2015, 3.2 million families had already started to use electricity in their residences representing additional 4.8% of the total number of residences (Villareal and Moreira, 2016). On the other hand, the Procel program was established to improve the country’s energy efficiency. The program started in 1984 with limited success, however, in 2001 it adopted minimum energy performance standards for several electric appliances improving its results (Nogueira et al., 2015).

3. LITERATURE REVIEW

3.1. International Literature

In the case of residential demand for electricity, most studies consider electricity demand in the context of household theory, i.e., households combine electricity and capital equipment to purchase a composite energy commodity (Narayan and Smyth, 2005). Ideally, an empirical model of residential electricity demand should be based on household demand theory and represent demand as a function of electricity price, real income, prices of substitute sources of energy, prices of electrical appliances, weather conditions, and other factors that could impact consumer

¹ (EPE, 2020).

² (EPE, 2020).

preferences (Amusa et al., 2009). In practice, most studies fail to reproduce ideal empirical specifications due to data constraints. A smaller number of studies use micro-level data seeking to find more variables that capture the household characteristics, such as Yoo et al. (2007), Filippini and Pachauri (2004) and Filippini (1999).

Although there is no consensus on which model is the best, there is a predominance of models that are estimated using univariate, multivariate or panel cointegration analysis (Dergiades and Tsoulfidis; 2011), Table 1. In this regard, the cointegration approach of Engle and Granger (1987) and mainly the multivariate cointegration procedures of Johansen (1988, 1991), Johansen and Juselius (1990) full information maximum likelihood technique, Pesaran and Pesaran (1997), Pesaran and Shin (1999) and Pesaran et al. (2001) have been widely used, as these models can provide a dynamic relationship between electricity consumption and its determinants (Amusa et al., 2009).

In short, these methods are autoregressive vectors restricted by an error correction term. This term represents the effects that deviations in the relation of cointegration produce on the dynamic behavior of the system in short run. The error-correcting term measures the proportion by which the long-run disturbance in the dependent variable is corrected in each short-run period, that is, the size of error-correction term measures the tendency of the dependent variable to return to its long-run equilibrium (Jamil and Ahmad, 2011).

Two major problems of ECM models are the lack of robustness to small sample sizes and the impossibility to test for long-run

relationship in the presence of a mix of stationary and non-stationary variables (Ziramba, 2008). These problems have prompted a revival in the application of the ARDL's cointegration procedure for dynamic modeling in many studies that use time-series data (Bentzen and Engsted, 2001). However, Fatai et al. (2003) suggest that ARDL and Johansen's approach give similar results both qualitatively and quantitatively.

As the cointegration models admit that the elasticity remains constant over time, some recent studies have applied time varying parameters models (TVP) based on Kalman filters such as Arisoy and Ozturk (2014), Chang et al. (2014), Inglesi-Lotz (2011), Wang and Mogi (2017). However, their conclusions differ about the constancy of elasticities. Table 1 presents a summary of energy studies and their long run income and price elasticities of electricity demand. These studies have examined the aggregate electricity demand or residential electricity demand and its main determinants in several countries.

3.2. Brazilian Literature

The first work to use econometric modeling to estimate the elasticities of electricity demand in Brazil was Modiano (1984). In this study, the author estimated a multiple regression model using the ordinary least squares method (OLS) with the purpose to analyze the sensitivity of consumption to economic activity and to electricity prices for all consumption classes in Brazil.

In order to avoid the spurious regression problem, Andrade and Lobão (1997) used for the 1st time an ECM model to explain the residential consumption of electricity in Brazil. As explanatory variables, the authors used the price of electricity, consumer

Table 1: Similar studies in the literature

Authors	Method	Price	Income	Period	Country
Bentzen and Engsted (1993)	ECM	-0.47	1.21	1960–1981	Denmark
Silk and Joutz (1997)	ECM	-0.48	0.52	1948–1990	United States
Fatai et al. (2003)	ECM	-0.55	1.24	1960–1999	New Zealand
	ARDL	-0.59	0.81	1960–1999	New Zealand
Hotledahl and Joutz (2004)	ECM	-0.15	1.57	1955–1996	Taiwan
Hondroyannis (2004)	ECM	-0.41	1.56	1986–1999	Greece
Narayan and Smyth (2005)	ARDL	[-0.54; -0.47]	[0.32;0.41]	1969–2000	Australia
De Vita et al. (2006)	ARDL	-0.34	1.27	1980–2002	Namibia
Halicioglu (2007)	ARDL	-0.52	0.70	1968–2005	Turkey
Zachariadis and Pashourtidou (2007)	ECM	-0.43	1.17	1960–2004	Cyprus
Dergiades and Tsoulfidis (2008)	ARDL	-1.07	0.27	1965–2006	United States
Ziramba (2008)	ARDL	-0.04	0.31	1978–2005	South Africa
Amusa et al. (2009)	ARDL	0.29*	1.67	1960–2007	South Africa
Inglesi (2010)	ECM	-0.56	0.42	1980–2005	South Africa
Dergiades and Tsoulfidis (2011)	ARDL	-0.606	0.795	1964–2006	Greece
Inglesi-Lotz (2011)	TVP	-0.075	0.794	1980–2005	South Africa
Jamil and Ahmad (2011)	ECM	0.07*	0.49*	1961–2008	Pakistan
Pourazarm and Cooray (2013)	ECM	-0.11*	0.58	1967–2009	Iran
Arisoy and Ozturk (2014)	TVP	-0.014	0.979	1960–2008	Turkey
Lim et al. (2014)	ECM	-1.00	1.09	1970–2011	South Korea
Ivy-Yap and Bekhet (2015)	ARDL	-0.942	1.417	1978–2013	Malaysia
Zaman et al. (2015)	ECM	-0.16	1.03	1972–2012	Pakistan
Wang and Mogi (2017)	TVP	-0.511	1.45	1989–2014	Japan
Campbell (2018)	ARDL	-0.82	0.26	1970–2014	Jamaica
Tiwari and Menegaki (2019)	TVP	-0.21	0.42	1975–2013	India
AlFalsh et al. (2020)	ECM	-1.22	0.04*	1972–2017	Kuwait
Othman and Hariri (2021)	ARDL	0.71*	0.38*	1980–2020	Malaysia

* Elasticities without statistical relevance

income and the stock of household appliances. After Andrade and Lobão (1997), other studies using ECM models were carried out with some different purposes, for instance to update the equation for Brazilian residential consumption, like Schmidt and Lima (2004) and Viana and Silva (2014), and to extend the application to other consumer segments or to replicate to Brazilian states and regions, like Mattos and Lima (2005); Mattos (2005) and Siqueira et al. (2006).

Furthermore, few studies estimated the elasticities of residential demand of electricity with other methods. With respect to studies with focus placed on some level of disaggregation, Uhr et al. (2017) used panel data for the twenty-seven Brazilian states to estimate short and long-run income and price elasticities of residential electricity consumption. Uhr et al. (2019) used household-level data to identify the demand elasticity of energy consumption made by Brazilian families. The authors were able to investigate heterogeneous effects across families using quantile regression (QR) analysis. The only study that considered spatial spillovers of electricity demand in the Brazilian setting was Cabral et al. (2020). These authors estimated income and price elasticities using the Dynamic Spatial Durbin model and concluded that taking into account the spatial dependence among Brazilian regions improve the goodness of fit. Table 2 presents a summary of empirical studies that provide elasticity estimates of residential electricity demand for Brazil.

It can be seen from Tables 1 and 2 that there is no consensus on the most appropriate methodology to be used for electricity modeling. To our knowledge, there is no study on electricity demand in Brazil that uses the bounds testing approach to cointegration within the autoregressive distributed framework.

4. METHODS AND MATERIALS

4.1. Empirical Specification and Data

The basic assumptions of the theoretical model of electricity demand are:

- i. The electricity demanded by residential consumers connected to the distribution network is fully met. It means that, in general or for most consumers, there is no problem of repressed demand so that energy supply is infinitely elastic. With this assumption, the quantity consumed can be used as a good approximation for the quantity demanded

- ii. Residential demand is influenced by three major variables: electricity price, consumer’s real income, and electrical equipment stock.

Studies on residential demand of electricity, either at local or international levels, traditionally make use of a Cobb-Douglas function, which has the following representation:

$$C_t = a_0 P_t^{a_1} Y_t^{a_2} E_t^{a_3} e^{\varepsilon_{1t}} \tag{1}$$

where C_t is the residential consumption of electricity at time t , a_0 is the drift term, Y_t the consumers’ real income, P_t the real average residential price of electricity, E_t the electrical equipment stock index, e the Neperian number, ε_{1t} the random error term, and, finally, a_1 , a_2 and a_3 are parameters to be estimated along with a_0 , and t is the time variable.

Because of the difficulty in obtaining electrical equipment stock data, some studies, such as Andrade and Lobão (1997) and Mattos and Lima (2005), assume an additional relationship for this variable, represented as follows:

$$E_t = b_0 Y_t^{b_1} P_t^{b_2} e^{\varepsilon_{2t}} \tag{2}$$

where P_t represents the average price of electrical equipment, ε_{2t} is the random error term, and b_0 , b_1 and b_2 are parameters to be estimated.

Substituting (2) into (1) yields an alternative specification for the electricity demand:

$$C_t = c_0 P_t^{c_1} Y_t^{c_2} P_t^{c_3} e^{\varepsilon_t} \tag{3}$$

where $c_0 = a_0 b_0^{a_3}$, $c_1 = a_1 + a_3 b_1$, $c_2 = a_2 + b_2 a_3$, $c_3 = b_2 a_3$ and $\varepsilon_t = \varepsilon_{1t} + a_3 \varepsilon_{2t}$. Therefore, (1) or (3) can be used as demand relations to be estimated. Brazilian studies usually adopt the last specification because of the absence of electrical equipment stock data in Brazil. By taking neperian logarithms in both sides of (3), we get the following econometric specification:

$$\ln C_t = \ln c_0 + c_1 \ln P_t + c_2 \ln Y_t + c_3 \ln P_t + \varepsilon_t \tag{4}$$

The goal is to estimate the parameters c_i ($i = 0, 1, 2, 3$). To carry out these estimations, we used yearly data on residential electricity demand from 1974 to 2016. Data sources were Eletrobrás SA, for residential consumption of electricity in GWh and average

Table 2: Empirical studies for Brazil

Authors	Method	Price	Income	Period	Level
Modiano (1984)	OLS	-0.40	1.13	1963–1981	Country
Andrade and Lobão (1997)	ECM	-0.05	0.21	1963–1995	Country
Schmidt and Lima (2004)	ECM	-0.09	0.54	1969–1999	Country
Mattos and Lima (2005)	ECM	-0.26	0.53	1979–2002	Minas Gerais
Siqueira et al. (2006)	ECM	-0.41	1.40	1970–2003	Northeast
Viana and Silva (2014)	ECM	-0.71	1.79	1975–2006	Country
Villareal and Moreira (2016)	OLS	-0.23	0.19	1985–2013	Country
Uhr et al. (2017)	GMM	-1.47; -0.62	0.32; 1.09	2004–2014	States
Uhr et al. (2019)	QR	-0.56; -0.46	0.20; 0.32	1998–1999 and 2008–2013*	Household**

* The authors used the Household Budget Survey (*Pesquisa de Orçamentos Familiares - POF* in Portuguese) for the 1998/1999 and the 2008/2013 surveys. ** The sample of households was drawn from the metropolitan area of São Paulo

price of electricity for residences in MWh/R\$; IPEA, for GDP, and FGV for IPA-OG's (wholesale price index – global supply) of home appliances³. Figure 2 display the series' behavior along the sample period.

As Figure 2a shows, electricity consumption had increased during the period of study except in 2001, when the electricity crisis mentioned before happened. The interesting part is that electricity consumption did not return to the previous level in the next year of 2002, when the restrictive policy of electricity consumption was removed. Therefore, residential consumers changed their habits permanently thereafter, continuing with consumption growth but at a lower trend level. In order to capture this structural change in the consumption's trend behavior, we added in the right side of equation (4) the dummy variable D_t , as suggested by Villareal and Moreira (2016), which is defined as:

$$D_t = \begin{cases} 0 & \text{for } t < 2001 \\ 1 & \text{for } t \geq 2001 \end{cases} \quad (5)$$

Expression (5) depicts D_t as a step function with break point at the year of 2001. Its purpose is to represent a negative intercept break due to the electricity crisis of 2001. The econometric specification that was ultimately adopted in this paper was:

$$\ln C_t = \ln c_0 + c_1 \ln P_t + c_2 \ln Y_t + c_3 \ln P_{e,t} + c_4 D_t + \varepsilon_t \quad (6)$$

4.2. Econometric Models

4.2.1. Error correction model

Estimation of equation (6) requires its variables $\ln C_t$, $\ln P_t$, and $\ln P_{e,t}$ to be stationary (Stock and Watson, 1989). If all three variables are non-stationary in levels, but stationary in first

differences, the Johansen cointegration test can be applied. Cointegration relationships are linear combinations of variables that are stationary and can be interpreted as long-term equilibrium relationships. Essentially, the ECM model is a vector autoregressive model (VAR) restricted with cointegration relationships through error correction mechanisms. These mechanisms represent the effects that deviations from equilibrium relations produce over the dynamic behavior of the system. ECM's representation associated with specification (6) is given by:

$$\Delta \ln C_t = \beta_0 + \sum_{i=0}^p \beta_{1i} \Delta \ln P_{t-i} + \sum_{i=0}^p \beta_{2i} \Delta \ln Y_{t-i} + \sum_{i=0}^p \beta_{3i} \Delta \ln P_{e,t-i} + \beta_4 \widehat{ect}_{t-1} + \beta_5 D_t + \varepsilon_t \quad (7)$$

where Δ is the difference operator, p is the number of lags, β_s is the s short-run elasticity and \widehat{ect}_{t-1} is the error correction term given by the cointegration relationship $\widehat{ect}_{t-1} = c_{t-1} - \alpha_0 - \alpha_1 p_{t-1} - \alpha_2 y_{t-1} - \alpha_3 [pe]_{t-1}$, where $\alpha_1, \alpha_2, \alpha_3$ can be interpreted as long-run elasticities.

where Δ is the difference operator, p is the number of lags, β_s is the s short-run elasticity and is the error correction term given by the cointegration relationship

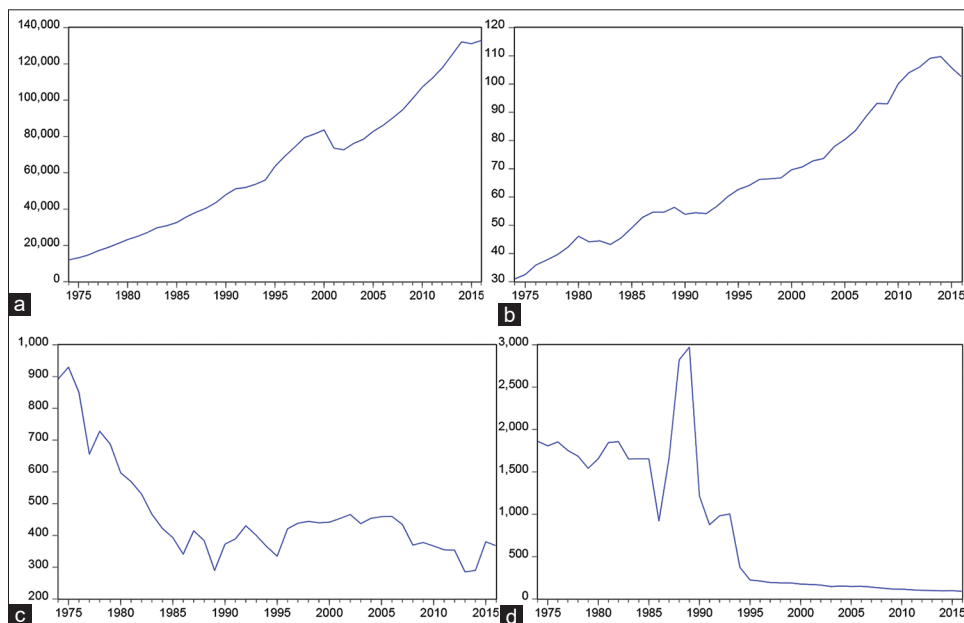
$$\widehat{ect}_{t-1} = c_{t-1} - \alpha_0 - \alpha_1 p_{t-1} - \alpha_2 y_{t-1} - \alpha_3 p_{e,t-1}, \text{ where } \alpha_1, \alpha_2, \alpha_3$$

can be interpreted as long-run elasticities.

4.2.2. ARDL cointegration model

ARDL models are a type of dynamic regression models with lagged terms for the dependent and independent variables. After the cointegration revolution consolidated in the late 1980s by Engle and Granger's (1987), Johansen's (1988), Johansen and Juselius' (1990) works, the ARDL models fell into disuse. However, they

Figure 2: Series' behavior



Source: (a) Residential consumption of electricity in GWh, (b) GDP (at 2016 prices), (c) Average price of residential electricity in MWh/R\$ (at 2016 prices) and (d) Wholesale price index of home appliances (at 2016 prices). Sources: Eletrobrás SA, IPEA and FGV

were revived in the late 1990s through the works of Pesaran and Pesaran (1997), Pesaran and Shin (1999) and Pesaran et al. (2001), who developed a consistent ARDL bounds testing procedure for checking the existence of a cointegration relationship.

This second cointegration method has the advantage, as compared Johansen’s testing approach, that it can be applied in the case where the variables are of mixed orders of integration, say, when there are both I(0) and I(1) variables (Pesaran and Pesaran, 1997).

Moreover, this technique is less susceptible to problems of endogeneity and usually provides unbiased long-run estimates with valid t-statistics (Narayan, 2005). The first step to implement the ARDL bounds testing procedure is to estimate by ordinary least squares (OLS) the following unrestricted error-correction model.

$$\begin{aligned} \Delta \ln C_t = & a_0 + \sum_{i=1}^p a_{1i} \Delta \ln C_{t-i} + \\ & \sum_{i=0}^p a_{2i} \Delta \ln P_{t-i} + \sum_{i=0}^p a_{3i} \Delta \ln Y_{t-i} \\ & + \sum_{i=0}^p a_{4i} \Delta \ln Pe_{t-i} + b_1 \ln C_{t-1} + b_2 \ln P_{t-1} + b_3 \ln Y_{t-1} \\ & + b_4 \ln Pe_{t-1} + b_5 D_t + \varepsilon_t \end{aligned} \tag{8}$$

The second step is to check whether the long run elasticities (b_s ; $s=1, \dots, 4$) are all zero or at least one is statistically significant. The preferred test used in the literature is the Wald test, which produces an F-statistic following a non-standard distribution. To assess the statistical significance of this test, Pesaran et al. (2001) propose two sets of critical values for a given significance level, the upper and lower critical bounds. These critical values can lead to three different conclusions: (i) If the F-statistic is smaller than the lower critical value, there is no cointegration; (ii) if it is between the upper and lower critical values, the test is inconclusive and (iii) if the F-statistic is above the upper bound it means that the series are cointegrated. However, Narayan (2005) argued that these critical values are inappropriate for small samples because they were generated by Pesaran et al. (2001) using a sample of 500 and 1000 observations. Therefore, we decided to use critical values established by Narayan (2005), because in this article we work with samples containing 30 to 80 observations.

Having identified the existence of a cointegration relationship, the third step is to select the best ARDL model specification. The model to be estimated is an ARDL (p, q_1, \dots, q_k), where p is the number of lags of the dependent variable and q_j is the number of lags of the j^{th} independent variable ($j = 1, \dots, k$). The ARDL (p, q_1, \dots, q_k) model applied to (6) can be written as follows:

$$\begin{aligned} \ln C_t = & c_0 + \sum_{i=1}^p c_{1i} \ln C_{t-i} + \sum_{i=0}^{q_1} c_{2i} \ln P_{t-i} + \\ & \sum_{i=0}^{q_2} c_{3i} \ln Y_{t-i} + \sum_{i=0}^{q_3} c_{4i} \ln Pe_{t-i} + b_5 D_t + u_t \end{aligned} \tag{9}$$

The long run elasticities founded in the cointegration relationship are non-linear functions of the coefficients in (9). To find this relationship we must equal all the variables in (8) to their respective contemporaneous versions to obtain the cointegration equation:

$$\ln C_t = d_0 + d_2 \ln P_t + d_3 \ln Y_t + d_4 \ln Pe_t + v_t \tag{10}$$

in which $d_0 = \frac{c_0}{1 - \sum_{i=1}^p c_{1i}}$, $d_m = \frac{\sum_{i=0}^{q_m} c_{mi}}{1 - \sum_{i=1}^p c_{1i}}$ for $m=2,3,4$ and

$$v_t = \frac{u_t}{1 - \sum_{i=1}^p c_{1i}}$$

In the last step, we estimate by OLS the short run coefficients:

$$\begin{aligned} \Delta \ln C_t = & h_0 + \sum_{i=1}^p h_{1i} \Delta \ln C_{t-i} + \sum_{i=0}^{q_1} h_{2i} \Delta \ln P_{t-i} + \\ & \sum_{i=0}^{q_2} h_{3i} \Delta \ln Y_{t-i} + \sum_{i=0}^{q_3} h_{4i} \Delta \ln Pe_{t-i} + h_5 \hat{v}_{t-1} + b_5 D_t + w_t \end{aligned} \tag{11}$$

where $h_5 \hat{v}_{t-1}$ is the error-correction term. The Akaike or Schwarz–Bayesian information criteria are recommended to help determine the ideal number of lags. Lastly, in order to ensure that the long-term relationship (10) is stable over time, it is necessary to check for the stability of long-run parameters. In this concern, we applied the tests based on the cumulative sum of recursive residuals (CUSUM) and the cumulative sum of squares (CUSUMQ), both proposed by Brown et al. (1975). Both tests are used to monitor whether a process is drifting away from its mean, in our case, whether the residuals are away from zero.

5. RESULTS AND DISCUSSION

5.1. Unit Roots and Cointegration Tests

As discussed before, it is necessary to check for the integration order of all variables before estimating ECM and ARDL models. In ECM models, cointegration is only possible if all variables have the same order of integration, while in ARDL models cointegration is possible when variables are of a mixed order of integration I(0) and I(1). To check for the integration order, we used three unit root tests: Augmented Dickey-Fuller (ADF), Dickey-Fuller - GLS (ERS) and Phillips-Perron (PP)⁴. These tests were chosen because they are efficient for small samples. The corresponding results are summarized in Table 3. All unit root tests indicate that the variables

4 For more details on these, Enders (2004), Elliot et al. (1996) and Perron and Ng (1996).

Table 3: Unit root tests

Variable	Include in test equation	ADF stat.	ERS stat.	PP stat.
C_t	Intercept and Trend	2.43	-1.32	-2.25
ΔC_t	Intercept	-3.38**	-3.19***	-3.38**
Y_t	Intercept and Trend	-3.38*	-2.64	-2.76
ΔY_t	Intercept	-4.52***	-4.49***	-4.49***
P_t	Intercept and Trend	-2.34	-1.92	-2.29
ΔP_t	Intercept	-6.95***	-6.71***	-7.03***
Pe_t	Intercept and Trend	-2.47	-1.94	-2.2
ΔPe_t	Intercept	-6.06***	-6.09***	-6.77***

(i) ***, ** and * indicates the absence of unit root at the significance levels 0.01, 0.05 and 0.10; (ii) In each test equation, the maximum number of lags was chosen by minimizing the Schwarz information criterion

are non-stationary in levels, but non-stationarity was rejected for first differences, suggesting that all variables are $I(1)$ at 5% level of significance. This result allows the possibility of cointegration in ECM and ARDL models.

To check whether there exists cointegration in the ECM model, we applied two tests proposed by Johansen (1988), the trace and the maximum eigenvalue tests. The results of these tests are reported in Tables 4 and 5. The statistical hypotheses of trace and max-eigen tests are slightly different, however the tests are equivalent when the rank of the co-integration matrix is null. Both trace and maximum eigenvalue tests indicated the presence of one cointegration relationship at the 1% significance level, which suggests the existence of a long-run relationship among the variables.

Having estimated (7) by means of OLS, cointegration in the ARDL structure was verified by the test proposed by Pesaran (2001). The result of this test is shown in Table 6 and suggest the existence of an equilibrium relationship only in the case where electricity consumption is the dependent variable at the 0.01 level of significance. The demand function notation was written as $F_c(C|Y,P,Pe)$, and does not change when the demand function is normalized with respect to independent variables (ex: $F_r(Y|C,P,Pe)$ for real average residential price demand function). We tested for other possible cointegration relationships to guarantee that the independent variables can be treated as long-run variables. The results shown in Table 6 indicate that there is only one cointegration relationship and all the independent variables can be treated as a long run determinants.

5.2. Estimation

Taking for granted the existence of a long-run relationship in both methods, we estimated (7) (ECM) and (11) (ARDL) considering a single cointegration relationship. We used the Schwarz Bayesian Criterion (SBC) for the selection of model's lag order. The final models were ECM(1) and ARDL (1,1,0,0). The short and long-run

elasticities resulting from the models, along with several model adjustment statistics are shown in Table 7. All variables showed the expected signals. The long-term coefficients of both models are remarkably similar, with the largest difference being 0.035, which suggested robust estimates.

As expected, all short-run elasticities were lower in absolute value than all long-run ones, however, they were not significant at the 5% level unlike long-run elasticities. As pointed out by Dergiades and Tsoulfidis (2008), the main reason for this fact is that in the short-run households demand is attached to the stocks of existing equipment, while in the long-run the stock of equipment changes. The lagged error correction term in the ECM model was -0.237 , while in the ARDL model it was -0.251 . Both were statistically significant at the 5% level with the expected negative sign, indicating that approximately 25% of the discrepancy between the effective value and the long-term value is corrected each year. Therefore, the residential sector would take approximately 4 years to adjust the residential demand for electricity to eventual shocks in the variables.

The long-run elasticities displayed expected signs, with a positive value for the income variable and negative values for the other two variables. Moreover, all of them are significant at the 5% level (except the price elasticity of the ECM model, which is significant only at the 6% level). The long-run income elasticities from ECM and ARDL models were 1.391 and 1.372, respectively. The estimated magnitudes are within the range obtained from previous studies for Brazil (Modiano, 1984; Villareal and Moreira, 2016) and other countries like Denmark (Bentzen and Engsted, 1993, 2001), New Zealand (Fatai et al., 2003), Greece (Hondroyiannis, 2004) and South Korea (Lim et al., 2014).

Both long-run price elasticities from ECM and ARDL confirm that electricity price play a minimal role in electricity consumption decisions of consumers, with estimated magnitudes of -0.298 and -0.263 respectively. The long-run electric equipment price index elasticities were practically the same, -0.160 for ECM and -0.157 for ARDL, indicating that this variable has little impact on electricity consumption.

Regarding the dummy variable D_r , the sign of the coefficients agreed with our premises, and both were statistically significant at the 5% level. The elasticities of ECM and ARDL were practically the same too, with $-0,089$ for ECM and -0.091 for ARDL. We notice these elasticities are less than that found by Villareal and Moreira (2016), of -0.221 .

Table 4: Johansen trace test for cointegration analysis

Rank	Eigenvalue	Trace	Critical	P-value	
H_0	H_1	statistic	value (1%)		
$r=0$	$r>0$	0.5596	68.4639	54.6815	0.0002
$r\leq 1$	$r>1$	0.4527	34.8425	35.4582	0.0120
$r\leq 2$	$r>2$	0.2180	10.1313	19.9371	0.2708
$r\leq 3$	$r>3$	0.0012	0.0480	6.6349	0.8264

The dummy variable of energy rationing was used as an exogenous variable in this test. The number of lags of the VAR model for the test was decided based on the Akaike information criterion

Table 5: Johansen max-eigen test for cointegration analysis

Rank	Eigenvalue	Max-Eigen	Critical	P-value	
H_0	H_1	statistic	value (1%)		
$r=0$	$r=0$	0.5596	33.6214	32.7153	0.0074
$r=1$	$r=1$	0.4527	24.7112	25.8612	0.0150
$r=2$	$r=2$	0.2180	10.0832	18.5201	0.2065
$r=3$	$r=3$	0.0012	0.0481	6.6349	0.8264

The dummy variable of energy rationing was used as an exogenous variable in this test. The number of lags of the VAR model for the test was decided based on the Akaike information criterion

Table 6: Bounds testing for cointegration in ARDL structure

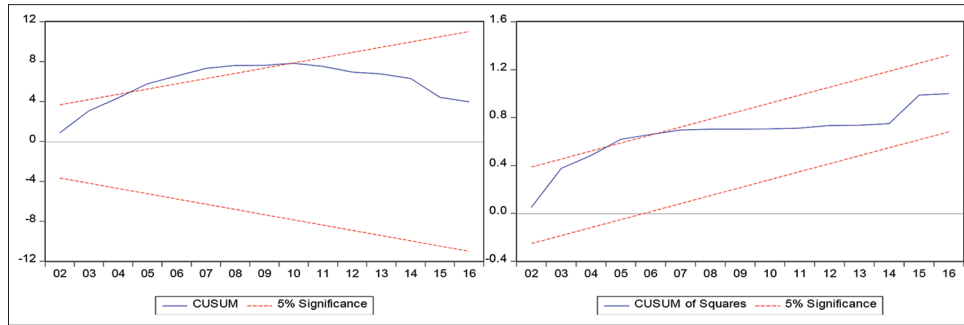
F-statistics	Critical value bounds (95%)		Critical value bounds (99%)	
	I (0)	I (1)	I (0)	I (1)
$F_c(C Y, P, Pe) = 38.75$	3.54	4.80	4.98	6.42
$F_y(Y C, P, Pe) = 3.35$				
$F_r(P C, Y, Pe) = 0.91$				
$F_{pe}(Pe C, Y, P) = 2.17$				

The critical values were obtained from Narayan (2005), p. 1988, Case III

All statistics of model adjustment from Table 6 suggest that the ARDL model provides a more robust estimation of elasticities than the ECM model. We performed Lagrange multiplier and Arch tests to verify serial correlation and heteroscedasticity of residues, respectively. The diagnostic tests indicate that the residuals are serially uncorrelated and homoscedastic at the 1% significance level.

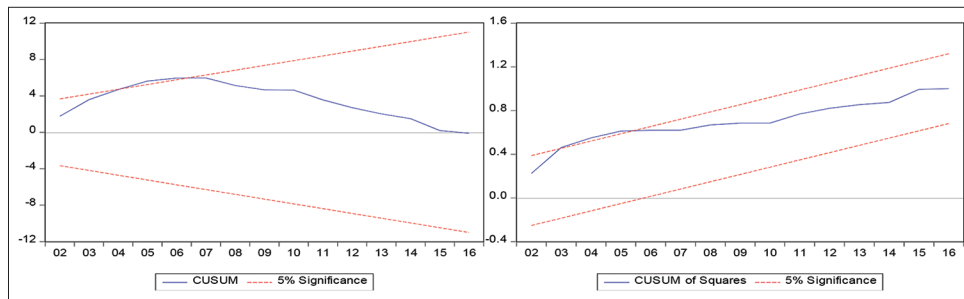
To check for the stability of the long-run elasticities, we applied the CUSUM and the CUSUMQ tests to the residuals of the models. Figures 3 and 4 display the results of CUSUM and CUSUMQ tests from ECM and ARDL models, respectively, with the dotted lines representing the critical upper and lower bounds at the 0.05 level of significance. The series of cumulative sum of recursive residuals and the cumulative sum of squares were generally within

Figure 3: CUSUM and CUSUMQ of ECM model cointegration



Source: The straight lines represent critical bounds at the 5% significance level

Figure 4: CUSUM and CUSUMQ of ARDL model cointegration



Source: The straight lines represent critical bounds at the 5% significance level

Table 7: Comparison of estimation results

Variables	ECM (1)			ARDL (1,1,0,0)		
	Coef.	T-statistic	P-value	Coef.	T-statistic	P-value
Short run elasticities						
Constant	0.094	5.62	0.0000	-	-	-
Dummy_ rationing	-0.089	-5.08	0.0000	-0.091	-10.199	0.0000
ΔC_{t-1}	-0.057	-0.37	0.7132	-	-	-
Δy_t	-	-	-	0.110	1.070	0.2919
Δy_{t-1}	-0.025	-0.15	0.8815	-	-	-
Δp_{t-1}	-0.004	-0.11	0.9129	-	-	-
Δpe_{t-1}	-0.009	-0.51	0.6127	-	-	-
EC_{1t-1}	-0.237	-4.41	0.0001	-0.251	-14.694	0.0000
Long run elasticities						
y_{t-1}	1.391	6.02	0.0000	1.372	6.29	0.0000
p_{t-1}	-0.298	-1.95	0.0579	-0.263	-2.43	0.0203
pe_{t-1}	-0.160	-4.18	0.0001	-0.157	-4.47	0.0001
Constant	8.247	-	-	-7.878	4.74	0.0000
Diagnostic tests						
	Statistic value		P-value	Statistic value		P-value
Serial Correlation	3.494		0.062	5.222		0.022
Heteroscedasticity	0.509		0.479	0.273		0.604
Model adjustment						
R^2 adjusted	0.56			0.73		
S. e. of regression	0.029			0.023		
Log-likelihood	90.11			100.66		
Akaike criterion	-4.05			-4.65		

Elaborated by authors

the 5% significance lines, suggesting that the residual variance is somewhat stable.

6. CONCLUSION AND POLICY IMPLICATIONS

This article described the determinants of the residential demand for electricity in Brazil over the period 1974-2016. The econometric specification assumes the demand for electricity depends on electricity price, per capita income, a price index of electrical equipment and a dummy variable that captures the effects of the 2001 Brazilian energy crisis. The Johansen' approach to cointegration testing and estimation and the bounds testing approach proposed by Pesaran (2001) both indicated a single cointegration relation among the variables and the electricity demand doesn't suffer with short run impacts. The long-run elasticities and the dummy variable coefficients of both models were similar, showing robustness.

The elasticity estimates obtained are consistent with the expectations about the signs of long-run parameters, and all are statistically significant at the 1% level. The long-run income elasticities were 1.391 (ARDL model) and 1.372 (ECM model), which are consistent with previous studies of Brazil and other countries. *Ceteris paribus*, a 1% increase in real income will result in nearly 1.39% increase in aggregate residential electricity consumption.

The long-run price elasticities were -0.298 (ECM model) and -0.263 (ARDL model), which confirms that changes in electricity price have minimum effect on the aggregate residential electricity consumption, as well as the long-run electric equipment price elasticities, -0.160 (ECM model) and -0.157 (ARDL model). The coefficient of the dummy variable representing the power generation crisis of 2001 has a negative sign, as expected, and is statistically significant in both models. The impact was -0.089 (ECM model) and -0.091 (ARDL model). Finally, the stability tests performed demonstrate that the long-run elasticities of residential electricity consumption in Brazil remained unchanged throughout the estimation period.

The stable function of the residential demand for electricity allows forecasting electricity demand for this sector at the national level. The estimated price and income elasticities imply that the residential demand for electricity in Brazil is price inelastic and income elastic. Such information can be valuable to policy makers in managing the supply of residential electricity and planning the expansion of the electricity system.

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