



Electricity Demand Forecasting of Value-at-Risk and Expected Shortfall: The South African Context

Bofelo Moemedi Masilo, Katleho Makatjane*

Department of Statistics, University of Botswana, Bostwana. *Email: makatjanek@ub.ac.bw

Received: 31 June 2024

Accepted: 07 November 2024

DOI: <https://doi.org/10.32479/ijeep.16995>

ABSTRACT

The energy landscape, particularly in the electricity sector, is characterised by a complex interplay of various factors that contribute to its inherent volatility. Electricity participants, including investors, regulators, consumers, and policymakers, are constantly seeking methods to better understand and manage the associated risks. The generalised extreme value (GEV) distribution with block minima is applied to model extreme losses on daily electricity demand in South Africa for the period of 1 April 2019 to February 13, 2024. The results of the estimated GEV gave a negative shape parameter implying that both winter and non-winter seasons extremes are correctly model a type III GEV distribution known as the Weibull distribution. When computing the VaR and ES, we found that VaR went as low as 8.74% while for ES had the lowest as 9.57%. Finally, the backtesting procedures further proved that the estimated risk measures are reliable as both the Kupiec and Chrisoffersen tests failed to reject the null hypothesis. In conclusion, the fitted GEV showed some reliance in capturing extremes losses for winter and non-winter returns. Lastly due to reliability of the models, risk analysts together with investors interested in the electricity sector should therefore adopt the procedures used to know the risk of their investment.

Keywords: Extreme Value Theory, Electricity Demand Load, Generalised Extreme Value, Volatility, South Africa

JEL Classifications: C1, C4, C5

1. INTRODUCTION

South Africa is heavily dependent on electricity mainly generated by coal production with an estimated power capacity of 90% (Mirzania et al., 2023). This is relatively high for a renewable energy resource, particularly for a large populated country such as South Africa with an estimated population of 62 million. This alone ought to cause an instability in the market of energy demand leading to issues of excess or demand deficiency. Demand deficiency according to Daniels (2022) arises from electricity users resorting to other means of energy generation like solar energy, biogas and many more. The energy sector is highly volatile and according to Šiml (2012), volatility clustering is a key risk factor that affects risk measures like value-at-risk (VaR). This means that extreme returns occur in clusters, highlighting the importance of managing volatility in risk management models. In addition, Chikobvu and Ndlovu (2024) showed that modelling volatility

gained popularity after Bellerslve (1986) has introduced GARCH (herein referenced Generalized Autoregressive Conditional heteroscedasticity) models; but Ardia et al. (2019) showed that the GARCH model is now a common risk management tool.

Nonetheless, Bauwens et al. (2014) underlined that the GARCH-family models provide biased estimates since their parameters are time-invariant, resulting in economic disasters caused by changes in economic conditions and investor expectations. Massacci (2017) further discovered that returns are fat-tailed, therefore risk strategies calculated during calm periods are ineffective during turbulent times. Obviously, this shifts the focus away from the middle of a conditional distribution but to the concept of tail risk. This can be difficult to measure in practice, because the distribution of returns during times of distress may not be adequately represented under standard parametric assumptions. Therefore, assumption of normality in GARCH models to predict volatility dynamics

is frequently challenged by empirical results; see for instance, Chikobvu and Ndlovu (2023a). This indicates that extreme losses are more likely to occur than those following a normal distribution. It is generally understood that the returns of emerging markets deviate significantly from normality, and this deviation is heavily impacted by the behaviour of large losses. Hence, Sigauke et al. (2014), advised that VaR and expected shortfall (ES) calculated using sample quantiles from a normal distribution when the data is fat-tailed are unreliable, henceforth a theory that considers these extremes is the so-called extreme value theory (EVT).

Beytell (2016) added that by utilising EVT methods in conjunction with risk measures yields reliable market risks, whereas Wei et al. (2013) observed that EVT is dependent on extreme observations to generate a distribution of random variable. This method of measuring risk is more efficient than simulating the whole distribution of a random variable. As a result, the current study uses EVT model to forecast the risk associated with extreme values in electricity demand resulting in economic shocks and financial instability in South Africa. Following a Beytell (2016), and Wei et al. (2013), we firstly estimate the GEV distribution to winter and non-winter electricity demand. In this way, our study hopes to assess the extreme trends for both winter and non-winter electricity and show a road mapping to extreme demand in both seasons. We further want to assess the risk associated with both season and show which season is likely to be affected by the losses through the application of VaR and ES.

2. METHODOLOGY AND ESTIMATION

This section mainly comprises of the methods used to model the VaR and expected shortfall of the electricity demand returns using the extreme value theory approach. The series under study are hourly electricity demand for the period of April 1, 2019 to February 13, 2024 and exhibit daily and weekly seasonalities. Hence, our model includes only daily seasonality. However, Smyl (2023) has emphasised that the daily seasonality (24 h values) is part of the weekly seasonality (168 h values), therefore the use of daily seasonalities in this study.

2.1. Generalized Extreme Value Distribution via Block minima

The generalized extreme value distribution is a family of continuous probability distributions developed within the extreme value theorem (Chikobvu and Chifurira, 2015). It consists of three different forms, the Gumbel (type I), the Fréchet (type II) and the Weibull distribution (type III), which are determined by the shape parameter. To illustrate the GEV via block minimum (BMM), we now, let X_1, \dots, X_n represent independent and identically distributed sample. The minimum values (gains) are now computed by

$$m_n = \min [X_1, \dots, X_n] = -M_n = -[\max[X_1, \dots, X_n]] \quad (1)$$

2.2. Block Maxima/Minima Method

One parametric method for EVT is the block maxima/minima method. It entails fitting the GEV to a specific set of maxima selected from a given data sample. As a result, we determined the ideal block size using the eight procedures listed below.

- Step 1: In this study, the daily electricity demand load is the data that is selected to function. We only allotted 25% of the generated data set to the test set of the study, in contrast to the 10% employed by Özari et al. (2019).
- Step 2: Blocks of data are divided into a minimum and maximum number. is the number of the block. Several blocks are generated from the daily electricity demand, starting with block five and going up to block fifty-nine. Over ninety trading days, this is intended to allow the risk measures to produce four exceptions or less.
- Step 3: For each block size, a minimum/maximum set of values is generated by computing the minimum/maximum value of each block. In this step, a data set is created for and the minimum/maximum values of each block are taken separately for these data sets.
- Step 4: For the general extremum value distribution, data is set appropriately. Using tests like Shapiro-Wilk, Anderson Darling, and Kolmogorov Smirnov tests, we are able to determine whether the fitted distribution of the blocks is appropriate.
- Step 5: The parameters of the optimal distribution for each value are determined.
- Step 6: The characteristics determined in step 5 as the over-value distribution or the number of observations segregated for testing with optimal distribution are used to create new variables, known as predictors, for each value.
- Step 7: The similarity between the test data and the k sets of estimated data is examined for each . If at all feasible, it is ideal for these two sets of data to be equal. The degree of resemblance between these two data sets can be determined in a variety of ways. In this study, we opted to employ the Pearson correlation to check for similarity; unlike Özari et al. (2019) who used the absolute difference technique to check for similarity.
- Step 8: The best block size is defined as the block with the highest correlation/similarity.

2.3. The Generalized Extreme Value Distribution

The GEV distribution can be denoted as $GEV(\mu, \sigma, \xi)$ and according to Gagaza et al. (2019), this distribution is given by

$$G(x; \mu, \sigma, \xi) = \exp \left\{ - \left[1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right]^{\frac{1}{\xi}} \right\} \quad (2)$$

Nonetheless, distribution (10) is valid $\left\{ x : \mu - \frac{\sigma}{\xi} < x < \infty \right\}$ and its probability density function (PDF) concerning Chikobvu and Ndlovu (2023b) is given by

$$f(x; \mu, \sigma, \xi) = \begin{cases} \exp \left(- \left(1 + \frac{\xi(x - \mu)}{\sigma} \right)^{\frac{-1}{\xi}} \times \frac{1}{\sigma} \left\{ 1 + \xi \left(\frac{x - \mu}{\sigma} \right) \right\} \right), & \xi \neq 0 \\ \frac{1}{\sigma} \exp \left[- \left(\frac{x - \mu}{\sigma} \right) - \exp \left(- \left(\frac{x - \mu}{\sigma} \right) \right) \right], & \xi = 0, \\ \mu \in \mathbb{R} \text{ and } \frac{\xi(x - \mu)}{\sigma} > 0 \end{cases} \quad (3)$$

Where μ, σ, ξ are the location, scale and shape parameters respectively.

Firstly the approach to choose between block maxima and block minima lied on the interest of the study, but before the choice of approach one should ensure that the observations are independent and identically distributed and divide the sample into non-overlapping subsamples (Chinhamu et al., 2015). Lastly the choice of the approach to fit the GEVD to lies on the nature of the data at hand and the interest of the study. Block maxima approach will be employed since the study's focus is on periods of peak energy demand (Ferreira and De Haan, 2015).

2.4. Maximum Likelihood Estimation

For parameter estimation purposes, we employ the method of maximum likelihood. It is favoured in statistical modelling for its three advantages: (1) it has desirable mathematical and optimality properties, (2) it could give a consistent approach to parameter estimation problems, (3) it is applicable in almost all popular statistical software packages as outlined by (Abdulali et al., 2022). Furthermore, it is not a difficult approach to use. Although it appears to be an excellent technique, its disadvantage is that it leads to underestimating of small sample sizes (Phoophiwfa et al., 2023), which can be deemed insignificant for this study because of large dataset being used. Abdulali et al. (2022) gave the steps to estimating the parameters of a probability distribution in three brief steps. The first step is obtaining the log likelihood function which is computed by

$$L_{(\mu, \sigma, \xi)} = \prod_{i=1}^n f_{\mu, \sigma, \xi}(x_i) \tag{4}$$

$$L_{(\mu, \sigma, \xi|x)} = \prod_{i=1}^n f(x_i | \mu, \sigma, \xi)$$

and, for GEVD, the result of will be

$$L(\mu, \sigma, \xi) = -n \log \sigma - \left(1 + \frac{1}{\xi}\right) \sum_{i=1}^n \log \left(1 + \xi \left(\frac{x_i - \mu}{\sigma}\right)\right) - \sum_{i=1}^n \left(1 + \xi \left(\frac{x_i - \mu}{\sigma}\right)\right)^{\frac{-1}{\xi}} \tag{5}$$

The second step is to take the natural log of the likelihood function and collecting terms involving the parameters μ, σ, ξ . The last step will be to differentiate $L(\mu, \sigma, \xi)$ and solve with respect to μ, σ and ξ to have

$$\frac{\partial}{\partial \mu} \{\log L(\mu)\} = 0$$

$$\frac{\partial}{\partial \sigma} \{\log L(\sigma)\} = 0$$

$$\frac{\partial}{\partial \alpha} \{\log L(\xi)\} = 0$$

(6)

2.5. Return Levels

The return level is a typical metric for extreme events. Chikobvu and Chifurira (2015) defines return levels to be the expected level

to be equal or exceeded on average once every interval of time (T) with a probability p. The following is the return level for a GEVD as per Saumi et al. (2016);

$$R_p^k = \mu + \frac{\sigma}{\xi} \left[\left(-\ln \left(1 - \frac{1}{k} \right) \right)^{-\xi} \right], \xi \neq 0 \tag{7}$$

But for $\xi = 0$, the model (7) becomes

$$R_p^k = \mu - \sigma \log \left(1 + \frac{1}{T} \right) \tag{8}$$

where T is the time interval in years.

2.6. Risk Measures

When dealing with energy generated mostly from renewable resources, it is worthwhile to know the amount of risk that might be attributed to energy usage that is above or below the normal threshold. To avoid any inconveniences from these occurrences, risk measures must be utilized to know the amount of units that might be needed in future. This section discusses the risk measures utilized in this study; VaR and ES.

2.7. Value-at-Risk

VaR is one commonly used measure of market risk, however it is not only limited to exploring the market risk, it can also manage all types of risk (Li, 2016). This makes it fit enough to explore even the risks relating to electricity demand. According to Li (2016), it is used to predict theoretically greatest loss portfolio for a particular time period for a specific situation. According to Chikobvu and Ndlovu (2023a) the VaR for calculating small probability p for the GEV distribution with maximum likelihood estimates is given as

$$VaR_p = \mu + \frac{\hat{\sigma}}{\hat{\xi}} \left\{ 1 - \left[-n \ln(1-p) \right]^{\frac{1}{\hat{\xi}}} \right\} \text{ for } \hat{\xi} \neq 0 \tag{9}$$

where n is the number of extrema. The value-at-risk is usually computed for the confidence levels between 95 and 99%. Nevertheless, Chinhamu et al. (2015), showed that, for a random variable X, with a distribution function F over a specified period of time, VaR can be defined as the pth quantile of F which is given as;

$$VaR_p = F^{-1}(1-p) \tag{10}$$

Where F⁻¹ known as the quantile function is the inverse of F.

2.8. Expected Shortfall

This measure in conjunction to VaR seeks to measure the potential loss incurred by a firm as a whole in an extreme event (Makatjane et al., 2021). As opposed to VaR, it was complimented for its ability to evaluate the losses beyond the VaR level and also being coherent as a risk measure. Makatjane (2022) noted that ES is sub-additive in addition to the mentioned advantages. According to Nadarajah et al. (2014), a generic computation for ES is given by

$$ES_p = \frac{1}{p} \left[E \left(XI \left\{ X \leq VaR_p(X) \right\} \right) + p VaR_p(X) - \right] \tag{11}$$

Due to it being an extension to the VaR, Acharya et al. (2017) outlined it to be the bank losses of confidence $1-\alpha$ denoted by $\Pr(R < -VaR^\alpha) = \alpha$. These authors further elaborated it to the losses a beyond the VaR level, which is given by

$$ES_\alpha = E[R | R \leq -VaR_\alpha] \tag{12}$$

2.9. Back Testing Risk Measures

The basic idea of back-testing risk measures is to ensure that the realised risk exposure is statistically in line with the expected one by comparing the actual losses with the reported VaR (Tsafack and Cataldo, 2021). It was further indicated that over violation of this is when the expected frequency of losses are above the above the reported VaR, which indicates that VaR was underestimated. There should not be a significant difference between the actual rate of violations and the expected one as this indicates inaccurate estimation of the VaR.

2.10. Kupiec Likelihood Ratio

Kupiec (1995) considered alternative statistical techniques that could be used to verify the accuracy of estimates of the tail values of the distribution of potential gains and losses for a portfolio of securities, futures, and derivative positions. These so-called reality tests were developed to determine the accuracy of risk exposure estimates generated by risk management models.

The strength of the model builds on its ability to predict precise risk estimates for appropriate capitalisation, hence Kupiec (1995) proposed the likelihood ratio statistic, LR_{UC} to accurately estimate the risk violation rate. This test also takes advantage of the fact that a good model should have its proportion of violations of risk estimates close to the corresponding tail probability. The procedure involves computing $x(\alpha)$ the number of times the reported returns are higher than the α risk estimate, i.e. $R_t > VaR(\alpha)$ and $R_t > ES(\alpha)$ and comparing the respective failure rate with α . Lok (2015) on the other hand stated that the rate of violation observed under the null hypothesis denoted as $\frac{S_n}{n}$ is equal to the expected violation rate denoted as $p = 1-\alpha$. Mwamba et al. (2017) further emphasised that the null hypothesis is derived as follows

$$H_0 : \hat{p} = \frac{S_n}{n}$$

$$H_a : \hat{p} \neq \frac{S_n}{n}$$

where, P , \hat{p} , S_n and n represent the theoretical proportion of violations, observed proportion of violations, frequency of violations and total sample size respectively. The assumption here is that the frequency follows the following binomial theorem $P(x) = \binom{n}{x} P^x (1-P)^{n-x}$. Hence, the Kupiec LR statistic is given by

$$LR_{UC} = 2 \ln \left[\frac{(1-p)^{n-x} p^x}{\left(1 - \frac{S_n}{n}\right)^{n-x} \left(\frac{S_n}{n}\right)^x} \right] \sim \chi^2_1 \tag{13}$$

Reject the null hypothesis if the observed probability value is greater than the calculated probability value and conclude that the model is not correct, implying that the risk computed from the risk model is unreliable. This means it can give false signals to risk managers.

2.11. Christoffersen Likelihood Ratio

Christoffersen is defined as a new backtesting tool that is based on the length of time between VaR violations. The test relies on the assumption that the violations are independent and identically distributed Bernoulli. The main insight is that where the VaR model is properly indicated for coverage, p , the expected duration of infringements should be a constant $1/\pi$ days. In this case, the Kupiec test was extended by the Christoffersen and Pelletier test (2004) to take account of the serial independence of violations (i.e. extreme clustering). The conditional precision for both ES and VaR should be tested when it is useful to control the inherent volatility cluster. The test provides a complete accuracy evaluation of both ES and VaR, according to Lee et al. (2012) it aims to determine if the violation indicator is $I^t = I_{\{L_t > VaR_{\frac{t}{\pi}}\}}$.

In general, the indicator variable is given by a binary variable denoted with 1 and 0, where the former indicates that a violation has occurred and the latter no violation has occurred. The fact that the test aims at establishing the dependence of violation, the notation η_{ij} is used to present the number of days where the j^{th} condition occurred given that the i^{th} condition previously occurred. Consequently, Table 1 presents a contingency table that presents possible outcomes.

The null hypothesis is that the violation indicator does not exhibit a first-order Markov property, i.e. $P(I^t=1 | I^{t-1}=0) = P(I^t=1 | I^{t-1}=1) = 1-\alpha$.

By defining $\pi_{ij} = \frac{\eta_{ij}}{\sum_j \eta_{ij}}$, where π_{ij} presents a violation probability

that occurs conditionally on state i at time $t-1$ such that $\pi_1 = \frac{\eta_{11}}{\eta_{11} + \eta_{21}}$, $\pi_2 = \frac{\eta_{22}}{\eta_{12} + \eta_{22}}$ and $\pi = \frac{\eta_{11} + \eta_{22}}{N}$, then the extended Kupiec LR test statistic is given by

$$LR_{CC} = -2 \ln \left(\frac{(1-\pi)\eta_{11} + \eta_{12}\pi\eta_{21} + \eta_{22}}{(1-\pi_1)\eta_{11}\pi_1\eta_{21} (1-\pi_2)\eta_{12}\pi_2\eta_{22}} \right) \tag{14}$$

In line with Papastathopoulos and Tawn (2013), expression 14 is an asymptotic chi-square distribution with one degree of freedom. Reject the null hypothesis if $\chi^2 > \chi^2_{\alpha, n-2}$ and conclude that the estimated risk measure is not a good measure for a specified risk.

3. EMPIRICAL ANALYSIS AND DISCUSSION

In this section of the study, we present the analysis and discussion of the results. These results are presented in tables and figures.

Table 1: Christoffersen contingency table

Exceptions	$I_{t-1} = 0$	$I_{t-1} = 1$	Total counts
$I_t=0$	η_{11}	η_{12}	$\eta_{11} + \eta_{12}$
$I_t=1$	η_{21}	η_{22}	$\eta_{21} + \eta_{22}$
	$\eta_{11} + \eta_{21}$	$\eta_{12} + \eta_{22}$	N

Following Makatjane (2022), we analyse losses on the winter and non-winter returns separately. Unless specified otherwise, the generalised extreme value distribution is implemented in R (R Core Team, 2021), RStudio (RStudio Team, 2022), evir (Pfaff and McNeil, 2018), rugarch (Ghalanos, 2020), ismev (Heffernan and Stephenson, 2018) and eva (Bader and Yan, 2020).

Figure 1 shows plots of the daily electricity demand. While 1c and 1d demonstrate that the data is non-normal, the kernel density plot in Figure 1d indicates that the distribution of the returns on electricity demand is leptokurtic demonstrating a kurtosis that is high above three. According to Pratiwi et al. (2019), this is an indication that the distribution of the returns do not have the same distribution as the normal, so it can be said that the returns has heavier tails than normal as reported in Figure 1d. Furthermore, it is noteworthy that in Figure 1a, seasonality is paired with certain positive and negative patterns. These moments of volatility clustering, according to Jacobo and Marengo (2020), are the result of events like the COVID-19 pandemic especially from the year 2020 to 2022. As Figure 1a shows, this electricity market in South Africa likewise has the most concentrated losses. Moreover, this figure represents a potential benefit when conditional heteroscedasticity is considered. Two important factors are highlighted here: the reason for weight loss; and the erratic nature of weight loss. The latter contends that irregular shocks in the actual energy sector have a greater influence on future volatility, whereas the former contends that downturn volatility follows these shocks rather than significant losses or gains. In Figure 1b the returns series are fairly stationary, around the zero mean, while high and non-constant fluctuations are noticeable, indicating volatility clustering and heteroscedasticity. Isolated extreme returns are visible, suggesting the extreme value distribution is relevant. The same results of volatility clustering are reported by Ndlovu and Chikobvu (2023) in their study of the generalised Pareto distribution model approach to comparing extreme risk in the exchange rate risk of bitcoin/us dollar and South African rand/us dollar returns.

Furthermore, Table 2 summarises the daily electricity demand returns. It is evident that the returns distribution are not normally distributed. As reported in Table 2, the kurtosis is above three implying that returns of electricity demand are leptokurtic indicating a fat tailed distribution. This results are in line with the one found by (De Domenico et al., 2023). The skewness is below zero implying negatively skewed distribution and this confirms the results reported in Figure 1(d) that returns are negatively skewed. This is an indication of asymmetric behaviour of the returns. Pal (2024) indicated that this entails the tail distribution is heavier than normal indicating returns distribution is leptokurtic. The Jarque-Bera, Shapiro-Wilk and Anderson-darling tests also attest to this non-normality of returns as all the three tests rejects the null hypothesis of normality with the calculated probability values that a <5% level of significance. The Ljung-Box test in Table 1, rejected the null hypothesis of no autocorrelation indicating the presence of autocorrelation. To address this issue, a block maxima approach is utilised to mitigate the autocorrelation problem because the BMM approach reduces this autocorrelation. Because the time series is divided into blocks (e.g., monthly, quarterly) and only the maximum/minimum value from each block. This approach was also employed by Chikobvu and Ndlovu (2023a) in their generalised extreme value distribution approach to comparing the riskiness of BitCoin/US Dollar and South African Rand/US Dollar Returns study. Additionally, the Augmented Dickey-Fuller (ADF) test confirms that the null hypothesis of a unit root is rejected at the 5% significance level, demonstrating that the return series for the electricity demand is stationary, as shown in Table 2.

Table 2: Descriptive statistics for the returns series

Variable	Mean	Median	Std. Dev	Skew	Kurt
Returns	0.00031	0.00089	0.0096	-3.9667	72.1220
Test	J-B test	S-W test	A-D test	ARCH test	Ljung-box
Statistic	1.3836	0.0009	0.0005	184.591	18.93019
P-value	0.001	0.001	0.001	0.001	0.001

Figure 1: Returns on daily electricity demand

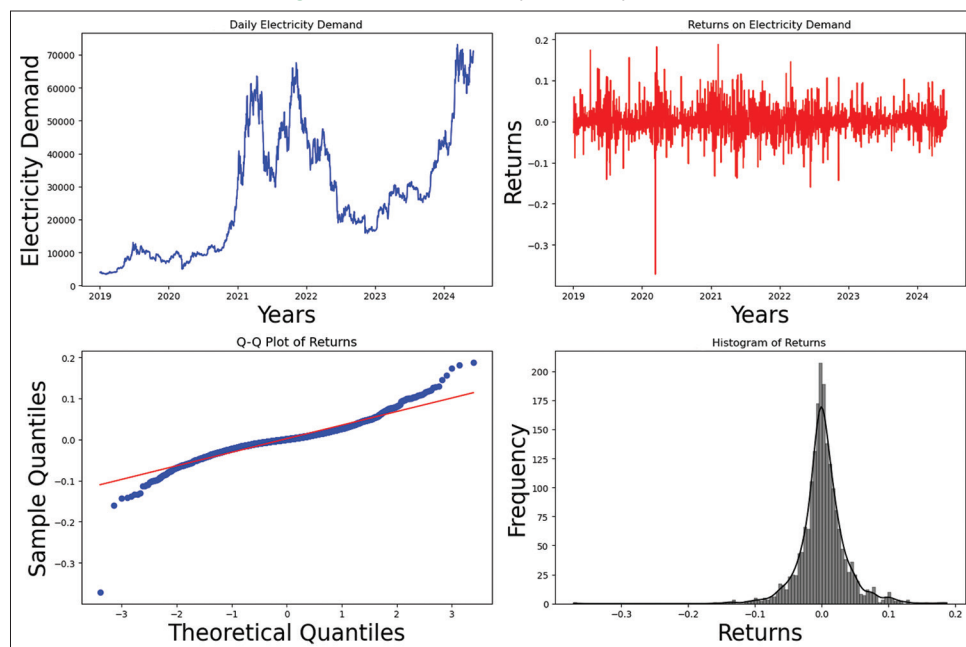


Table 3: Maximum likelihood estimates for GEV

Season	Block size	Maxima	$\hat{\mu}$	$\hat{\sigma}$	$\hat{\xi}$
Winter	5	315	9.592 (0.01)	0.2828 (0.001)	-1.0885 (0.001)
Non-winter	5	899	9.571 (0.02)	0.2878 (0.02)	-0.7577 (0.062)

NB: The values in () are the standard error of the parameter estimates

3.1. Estimation of GEV Model

Unlike Makatjane (2022) who used the generalised Pareto distribution, we fit the GEV distribution to winter and non-winter season's returns separately. Ndlovu and Chikobvu (2023) extracted monthly period minima/maxima in their study. But, with the current study, we use 5 day block minima and this resulted in a total of 315 block minimas for winter season and a total of 899 block minimas for non-winter season. The reason behind much variation in block maxima size is because winter seasons was taken to be the months June, July and August and the rest of the months were taken to be non-winter. The monthly categories was done looking at the weather similarities for months taken to be winter season and those taken to be non-winter. Table 3 summarises the estimated parameters of the GEV model together with their standard error estimates.

The shape parameters reported in Table 3 for both the winter and non-winter seasons are negative. This is an indication that the returns distributions takes that of a Weibull distribution. (Liu and Hong, 2022), Gagaza et al. (2019) and Sigauke et al. (2014) also reported a negative shape parameter estimates. The contrast is in an empirical analysis of Chan (2016). This author reported a positive shape parameter in their study, which indicates a Fréchet distribution.

Nevertheless, (Pratiwi et al., 2019) further indicated that the location parameter on the other side indicates the data centre point. The location parameter of non-winter returns being greater than that of the winter returns indicate that there is a high probability for the occurrence risk of extreme events as per Pratiwi et al. (2019) implying higher risk of excess demand or low demand of electricity. Finally the left endpoint for both seasons are computed as $\hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}} = 9.852$ for winter seasons and $\hat{\mu} - \frac{\hat{\sigma}}{\hat{\xi}} = 9.95083$ and it implies that for any degree losses above 5%, the likelihood of any further decrease in winter and non-winter electricity demand is minimal.

The analysis of minima provides insights into periods of low electricity demand, which can be critical for operational planning. The higher number of minima in non-winter (899) compared to winter (315) suggests that there may be more frequent low-demand periods during non-winter months. The location estimates indicate that average low demand is slightly lower in winter than in non-winter, suggesting that Eskom may experience more consistent low demand in summer. This could allow Eskom to optimize generation schedules and reduce operational costs during these low-demand periods (Khamrot et al., 2024). Given the potential for increased low-demand periods, Eskom might explore energy storage solutions to balance supply and demand effectively. By storing excess energy generated during low-demand periods,

Eskom can ensure a stable supply during peak times or unexpected demand fluctuations. As with maxima, understanding minima is crucial for compliance with regulations regarding emissions and sustainability. By managing low-demand periods effectively, Eskom can reduce waste and improve its overall environmental impact.

3.2. Goodness of Fit Test

After model estimation, a goodness of fit (GoF) test is assessed. The Anderson-Darling, Jarque-Bera and Shapiro-Wilk tests are used the results of these tests are shown in Table 4. Nonetheless, Stephens (1977) recommended these for the GoF for extreme value distributions. Chikobvu and Chifurira (2015) and Maposa et al. (2016) also used these tests for testing GoF for both GEV and GPD models in their studies. The results of these three tests fails to reject the null hypothesis of normality; hence the study concludes that returns on daily electricity demand are well fitted by the GEV distribution. This is met by observing high probability values that are above 10% level of significance.

The residual quantities are exponentially distributed. Most of them lie on straight lines, which shows that the GEV is well-suited to this return index. These results are reported in Figure 2. The results reported in Chinhamu et al. (2015) are similar to the one found in this study. The more block sizes, the more the GEV distribution fits the data well. Therefore, the Fisher theorem only applies if the block size $n \rightarrow \infty$ (Fisher and Tippett 1928). Although the estimate is near zero, the estimated curve is nearly quadratic. The curve also provides an adequate representation of the empirical estimates, particularly after taking into account sampling variability. Finally, the density estimate corresponding to the histogram for the data appears to be consistent. Thereby, the fitted GEV is supported by the four diagnostic plots.

3.3. Return Levels Estimates

Table 5 indicates the return periods for 2, 3, 5, and 10 years for both winter and non-winter seasons. For non-winter, the expected tail-related loss of 9.662937%, 9.758999% 9.828758% and 9.881625 at short (2 years), medium (3 and 5 years), and long (10 years) terms, respectively, are greater than the expected tail-related losses of 9.677839%, 9.754928%, 9.801420%, 9.829760 for winter season. Hence a long position (holding a unit of non-winter electricity hoping to demand electricity on a later date at a higher price) is recommended rather than a short position (selling a unit of winter season and buying it back at a later date) since there is a higher chance of realising a loss in the long run when holding a non-winter electricity demand.

With higher expected minima during winter, Eskom may need to adjust its infrastructure and capacity management strategies

to ensure reliability during these periods. This could involve maintaining flexibility in generation capacity or enhancing demand response programs to manage low demand efficiently. Eskom can use these insights to develop mitigation strategies against potential financial losses associated with lower-than-expected demand during extreme events. Knowing when low-demand periods are likely to occur can help Eskom better integrate renewable energy sources into its grid management strategies. This could enhance overall efficiency and sustainability by ensuring that

renewable generation aligns with expected demand patterns (Fase et al., 2024).

3.4. Estimation of Risk Measures

Two risk measures, VaR, ES used in this study compute the risk for electricity demand load are presented in Table 6. Estimation of this measures utilised EVT as a form of estimation. Value-at-Risk is a high quantile of the loss distribution (Chikobvu and Ndlovu, 2023a). The VaR was evaluated at 95 and 99% respectively. At 95% interval, VaR is 8.99% which indicates that 95% of times the extreme losses are expected not to exceed 8.99% which is a similar interpretation for all the estimated risk values. Non-winter returns turned to be slightly at a higher risk of having a higher risk of experiencing unstable electricity load demand as compared to winter seasons when evaluated at 95% interval. At 99% confidence interval, winter returns turned to be at a higher risk than the non-winter returns. Another risk measure expected shortfall showed the expected risks that lie around 9.6%, on average expected shortfall showed risks slightly higher than the value at risk. It must be considered that value at risk tends to underestimate the risks but since the risks do not deviate much, it may be concluded that the highest risk at both 99 and 95% is 9.6% for both winter and non-winter seasons. In summary these results implies that for both 95% and 99% intervals, non-winter is more risky than winter season. Which is the surprising because in winter electricity is demand is high indicating that high losses are expect in this season.

The computed risk measures provide Eskom with valuable insights into potential daily losses in electricity consumption. Understanding these risks helps in developing strategies to mitigate financial impacts during periods of low demand. The differences in VaR and ES between winter and non-winter indicate varying risk profiles. For instance, winter has a higher VaR (9.34 at CI 0.99) compared to non-winter (8.74 at CI 0.99), suggesting more significant potential losses during winter months. By anticipating potential losses, Eskom can allocate resources more effectively and develop contingency plans to manage financial risks. According to Panda et al. (2023), the insights from VaR and ES can guide operational strategies, particularly in demand-side management. Eskom may consider implementing demand response programs to mitigate risks associated with unexpected low-demand periods. Understanding financial risks associated with electricity consumption can also aid in compliance with regulatory requirements regarding financial stability and operational reliability.

3.5. Backtesting of Procedures

The summary of backtesting results of ES and VaR shown in Table 7 were tested at 95 and 99% for both winter and non-winter

Table 4: Goodness of fit test for GEV distribution

Tests	Winter	Non-Winter
A-D		
Statistic	0.415	0.551
P-value	0.335	0.156
S-W		
Statistic	0.062	0.074
P-value	0.362	0.247
J-B		
Statistic	3.02	4.28
P-value	0.237	0.125

Table 5: Return levels estimates

Period	2-year	3-year	5-year	10-year
Winter	9.677839	9.754928	9.801420	9.829760
Non-winter	9.662937	9.758999	9.828758	9.881625

Table 6: Computation risk measures on daily losses in electricity consumption

Season	CI	VaR	ES
Winter	0.95	8.99	9.57
	0.99	9.34	9.28
Non-winter	0.95	9.08	9.58
	0.99	8.74	9.59

Figure 2: Diagnostics GEV model

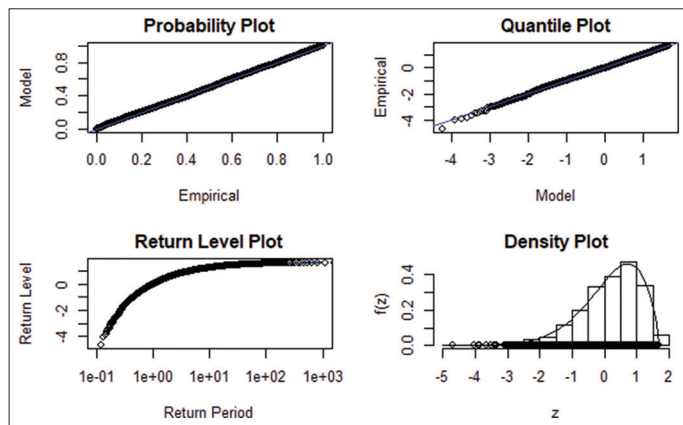


Table 7: Backtesting of VaR and ES

P-values for Kupiec test				P-values for Christoffersen test			
Risk measure	Level	0.95	0.99	Risk measure	Level	0.95	0.99
VaR	Non-Winter	0.517	0.378	VaR	Non-Winter	0.207	0.885
ES	Non-Winter	0.919	0.877	ES	Non-Winter	0.865	0.987
Risk Measure	Level	0.95	0.99	Risk Measure	Level	0.95	0.99
VaR	Winter	0.529	0.902	VaR	Winter	0.801	0.739
ES	Winter	0.668	0.872	ES	Winter	0.701	0.885

seasons. The null hypothesis is that the model accurately evaluates winter and non-winter returns risk. The values in the table all deemed dependable for both Kupiec and Christofferson tests since they are all $>10\%$. This implies that we fail to reject the null hypothesis and conclude that the model accurately predicts the related losses for both tests. For non-winter season, ES proved to be more consistent when evaluated at both 95% and 99% for both tests due to larger p-values than the VaR. Similarly, for winter returns, when evaluated at 95% using Kupiec test, ES turned to be slightly more adequate than VaR with ES being 0.668 and VaR 0.529. Conversely, when both were evaluated at 99% confidence interval, VaR turned to be more adequate than ES. The Christofferson test when employed at 95% confidence interval indicated VaR to be more appropriate in estimating the return losses of winter seasons. At 99% confidence, ES accurately predicted losses than VaR. Generally ES could be a better risk metric to VaR looking at the results from the table which is similar to Götz and Laitenberger (2024)'s findings when using multiple tests.

4. CONCLUSION AND RECOMMENDATIONS

The extreme value theory was employed in this work to estimate the risk of electrical load demand returns using VaR and ES. This came about as a result of discovering the measures' potential for risk assessment and their track record of good performance across a variety of disciplines, which suggested that because the data contained extremes, it may even be beneficial for returns on electricity demand loss. In order to ease risk assessment using EVT, a 5-day block maximum of 315 for winter returns and 899 for non-winter returns was fitted using the extreme value distribution to capture extreme quantiles. Utilising EVT was primarily motivated by its shown capacity to capture extremes in a variety of research domains, one of which was a study by Riaman et al. (2023) on VaR estimation in stock investment of insurance companies and Orsini et al. (2020) on Large scale road safety evaluation. Additionally, failing to record extremes might have yielded erroneous estimation findings since they would have been disregarded. The method was further suggested by normality checks together with proved adequacy by the GoF tests.

The results from the fitted GEV model were then used to estimate VaR and ES. Winter returns estimates of risk at 95 and 99% confidence were discordant, VAR showed a risk of 8.99446% while ES shortfall showed a risk of 9.567814% which both indicated the greatest percentage amount of change in the levels of demand. In simple terms if Eskom is to make any investment in electricity, it should be ready for that amounting change in electricity demand. At 99% confidence ES showed less risk than the VaR of 9.579067 compared to 9.842173. For non-winter seasons, ES proved to be the right wing in risk estimation. Evaluated at both 95 and 99% confidence, ES had less estimates than VaR. In general, it may be concluded that winter seasons are more conservative than non-winter since the 95% confidence has a smaller margin of error. To ensure the adequacy of the procedure, the results were backtested using the Kupiec and Christoffersen likelihood ratio tests. All the tests failed to reject the null hypothesis that the model accurately evaluates the winter and non-winter returns risk.

The proved accuracy of the procedure should motivate the electricity sector to consider applying a similar procedure to make informed decisions when it comes to related investments. The findings are not only beneficial to power entities, individuals and industries undertaking major investments may refer to the findings to avoid inconveniences by power cuts that are influenced by changing levels of demand. In conclusion the findings of this study together with the procedure may be adopted by any relevant entities for profitable and informed investments.

REFERENCES

- Abdulali, B.A.A., Abu Bakar, M.A., Ibrahim, K., Mohd Ariff, N. (2022), Extreme value distributions: An overview of estimation and simulation. *Journal of Probability and Statistics*, 2022(1), 5449751.
- Acharya, V.V., Pedersen, L.H., Philippon, T., Richardson, M. (2017), Measuring systemic risk. *The Review of Financial Studies*, 30(1), 2-47.
- Ardia, D., Bluteau, K., Boudt, K., Catania, L., Trottier, D.A. (2019), Markov-switching GARCH models in R: The MSGARCH package. *Journal of Statistical Software*, 91(4), 1-38.
- Bader, B., Yan, J., Bader, M.B. (2020), Package "eva". Available from: <https://cran.r-hrc.org/web/packages/eva/eva.pdf> [Last accessed on 2024 Mar 24].
- Bauwens, L., De Backer, B., Dufays, A. (2014), A Bayesian method of change-point estimation with recurrent regimes: Application to GARCH models. *Journal of Empirical Finance*, 29, 207-229.
- Bellarslve, T. (1986), Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31(3), 307-327.
- Beytell, D. (2016), The Effect of Extreme Value Distributions on Market Risk Estimation. South Africa: University of Johannesburg.
- Chan, K.S. (2016), Statistical Modelling of Extreme Values for Dependent Variables. (Doctor of Philosophy PhD), The Hong Kong University of Science and Technology, Hong Kong. Available from: <https://hdl.handle.net/1783.1/82341>
- Chikobvu, D., Chifurira, R. (2015), Modelling of extreme minimum rainfall using generalised extreme value distribution for Zimbabwe. 111(9-10), 1-8.
- Chikobvu, D., Ndlovu, T. (2023a), The generalised extreme value distribution approach to comparing the riskiness of bitcoin/US dollar and South African rand/US dollar returns. *Journal of Risk Financial Management*, 16(4), 253.
- Chikobvu, D., Ndlovu, T. (2023b), The generalised extreme value distribution approach to comparing the riskiness of BitCoin/US dollar and South African Rand/US dollar returns. *Journal of Risk and Financial Management*, 16(4), 253.
- Chikobvu, D., Ndlovu, T. (2024), Statistical analysis of the BitCoin and South African rand exchange rates risks when the tails are somewhat heavy. *Journal of Statistics Application and Probability*, 13(5), 1411-1429.
- Chinhamu, K., Huang, C.K., Huang, C.S., Hammujuddy, J. (2015), empirical analyses of extreme value models for the South African Mining Index. *South African Journal of Economics*, 83(1), 41-55.
- Daniels, N. (2022), See how Much Stage 6 Costs SA. Available from: <https://www.iol.co.za/capetimes/news/see-how-much-stage-6-costs-sa-4456ef54-b30c-4350-ab44-77bc9b30e4f6> [Last accessed on 2024 Jan 23].
- De Domenico, F., Livan, G., Montagna, G., Nicosini, O. (2023), Modeling and simulation of financial returns under non-Gaussian distributions. *J Physica A: Statistical Mechanics its Applications*, 622, 128886.

- Ferreira, A., De Haan, L. (2015), On the block maxima method in extreme value theory: PWM estimators. *The Annals of Statistics*, 2015, 276-298.
- Fisher, R.A., Tippett, L.H.C. (1928), Limiting Forms of the Frequency Distribution of the Largest or Smallest Member of a Sample. In: Paper presented at the Mathematical Proceedings of the Cambridge Philosophical Society, United Kingdom.
- Gagaza, N., Nemukula, M.M., Chifurira, R., Roberts, D.J. (2019), Modelling non-Stationary Temperature Extremes in KwaZulu-Natal using the Generalised Extreme Value Distribution. In: Paper presented at the Annual Proceedings of the South African Statistical Association Conference, Port Elizabeth.
- Ghalanos, A. (2020), Introduction to the Rugarch Package. (Version 1.3-1). Manuscript. Available from: <https://cran.r-project.org/web/packages/rugarch> [Last accessed on 2024 Mar 24].
- Götz, P., Laitenberger, J. (2024), Forecasting Expected Shortfall with Multiple Quantiles. Available from: <https://ssrn4684001> [Last accessed on 2024 Apr 10].
- Heffernan, J.E., Stephenson, A.G. (2018), ISMEV: An Introduction to Statistical Modeling of Extreme Values [code]. CRAN. Available from: <https://cran.r-project.org/package=ismev> [Last accessed on 2024 Mar 24].
- Jacobo, A.D., Marengo, A. (2020), Are the business cycles of Argentina and Brazil different? New features and stylized facts. *Paradigma Económico. Revista de Economía Regional y Sectorial*, 12(2), 5-38.
- Kupiec, P.H. (1995), Techniques for Verifying the Accuracy of Risk Measurement Models. Vol. 95. Division of Research and Statistics, Division of Monetary Affairs. Aluva: Federal.
- Li, W. (2016), Value at Risk (VaR) and Its Calculations: An Overview. Rolla: Missouri University of Science Technology.
- Liu, Y., Hong, H. (2022), Estimating quantiles of extreme wind speed using generalized extreme value distribution fitted based on the order statistics. *Wind and Structures*, 34(6), 469-482.
- Lee, D., Li, W.K., Wong, T.S.T. (2012), Modelling insurance claims via a mixture exponential model combined with a peaks-over-threshold approach. *Insurance: Mathematics and Economics* 51(3), 538-550.
- Makatjane, K., Moroke, N., Munapo, E. (2021), Predicting the Tail Behaviour of Financial Times Stock Exchange/Johannesburg Stock Exchange (FTSE/JSE) Closing Banking Indices: Extreme Value Theory Approach. In: Adigüzel Mercangöz, B., editor. *Handbook of Research on Emerging Theories, Models, and Applications of Financial Econometrics*. Cham: Springer International Publishing. p31-64.
- Maposa, D., Cochran, J., Lesaoana, M. (2016), Modelling non-stationary annual maximum flood heights in the lower Limpopo River basin of Mozambique. *Jamba: Journal of Disaster Risk Studies*, 8(1), 1-9.
- Massacci, D. (2017), Tail risk dynamics in stock returns: Links to the macroeconomy and global markets connectedness. *Management Science*, 63(9), 3072-3089.
- Mirzania, P., Gordon, J.A., Balta-Ozkan, N., Sayan, R.C., Marais, L. (2023), Barriers to powering past coal: Implications for a just energy transition in South Africa. *Journal of Energy Research Social Science*, 101, 103122.
- Mwamba, J.W.M., Hammoudeh, S., Gupta, R. (2017), Financial tail risks in conventional and Islamic stock markets: A comparative analysis. *Pacific-Basin Finance Journal*, 42, 60-82.
- Nadarajah, S., Zhang, B., Chan, S. (2014), Estimation methods for expected shortfall. *Quantitative Finance*, 14(2), 271-291.
- Ndlovu, T., Chikobvu, D. (2023), The generalised pareto distribution model approach to comparing extreme risk in the exchange rate risk of BitCoin/US Dollar and South African Rand/US Dollar Returns. *Risks*, 11(6), 100.
- Orsini, F., Gecchele, G., Gastaldi, M., Rossi, R. (2020), Large-scale road safety evaluation using extreme value theory. *IET Intelligent Transport Systems*, 14(9), 1004-1012.
- Özari, Ç., Eren, Ö., Saygin, H. (2019), A new methodology for the block maxima approach in selecting the optimal block size. *Tehnički Vjesnik*, 26(5), 1292-1296.
- Pal, D. (2024), The distribution of commodity futures: A test of the generalized hyperbolic process. *Applied Economics*, 56(15), 1763-1783.
- Pfaff, B., McNeil, A., Stephenson, A. (2018), evir: Extreme Values in R. R Package Version. p1-7. Available from: <https://www.pfaffikus.de/rpacks/evir/files/manual-evir.pdf> [Last accessed on 2024 Mar 24].
- Phoophiwfa, T., Laosuwan, T., Volodin, A., Papukdee, N., Suraphee, S., Busababodhin, P. (2023), Adaptive parameter estimation of the generalized extreme value distribution using artificial neural network approach. *Atmosphere*, 14(8), 1197.
- Pratiwi, N., Iswahyudi, C., Safitri, R. (2019), Generalized extreme value distribution for value at risk analysis on gold price. *Journal of Physics: Conference Series*, 1217, 012090.
- R Core Team. (2021), R: A Language and Environment for Statistical Computing. R Foundation for Statistical Computing, Vienna. Available from: <https://www.r-project.org>
- RStudio Team. (2022), RStudio: Integrated Development for R. RStudio. PBC. Available from: <https://www.rstudio.com>
- Riaman, R., Octavian, A., Supian, S., Sukono, S., Saputra, J. (2023), Estimating the value-at-risk (VaR) in stock investment of insurance companies: An application of the extreme value theory. *Decision Science Letters*, 12(4), 749-758.
- Saumi, T.F., Wigena, A.H., Djuraidah, A. (2016), Return level value modeling of rainfall and GCM for extreme rainfall prediction in indramayu regency. *Global Journal of Pure Applied Mathematics*, 12(1), 1151-1159.
- Sigauke, C., Makhwiting, R.M., Lesaoana, M. (2014), Modelling conditional heteroskedasticity in JSE stock returns using the generalised pareto distribution. *African Review of Economics and Finance*, 6(1), 41-55.
- Šiml, J. (2012), Extreme Value Theory: Empirical Analysis of Tail Behaviour of GARCH Models. Czechia: Univerzita Karlova, Fakulta Sociálních Věd.
- Smyl, S., Dudek, G., Peřka, P. (2023), Forecasting cryptocurrency prices using contextual ES-adRNN with exogenous variables. In: Mikyřka, J., de Mulatier, C., Paszynski, M., Krzhizhanovskaya, V.V., Dongarra, J.J., Sloot, P.M., editors. *Computational Science - ICCS 2023. ICCS 2023. Lecture Notes in Computer Science*. Vol. 14073. Cham: Springer.
- Stephens, M.A. (1977), Goodness of fit for the extreme value distribution. *Biometrika*, 64(3), 583-588.
- Tsafack, G., Cataldo, J.J. (2021), Backtesting and estimation error: Value-at-risk overviolation rate. *Empirical Economics*, 61(3), 1351-1396.
- Wei, Y., Chen, W., Lin, Y. (2013), Measuring daily value-at-risk of SSE index: A new approach based on multifractal analysis and extreme value theory. *Physica A: Statistical Mechanics*, 392(9), 2163-2174.