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Quantile Regression Model for Peak Load Demand Forecasting with Approximation by Triangular Distribution to Avoid Blackouts#

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ABSTRACT

Peak load demand forecasting is a key exercise undertaken to avoid system failure and power blackouts. In this paper, the next day's peak load demand is forecasted. The challenge is to estimate a model that is capable of preventing underprediction of the peak load demand: In other words, a model that is competent in forecasting the upper bound of the peak demand to avoid the risk of power blackouts. First, quantile regression is performed to generate forecasts of the daily peak load demand. Then, peak demand forecasts are locally approximated by triangular distribution to generate the upper bound of the peak demand. The forecasted upper bounds are compared with the actual electricity demand. The proposed method succeeds in avoiding underprediction of the peak load demand and thus the risk of power blackouts.

Keywords: Electricity Peak Demand, Quantile Regression, Triangular Distribution, Blackouts
JEL Classifications: Q47, C21

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1. INTRODUCTION

The peak demand is the highest load observed during a (short) unit of time. Forecasting the peak load demand is a fundamental task for ensuring the availability of sufficient supply. Peak load demand forecasts are critical, since electricity is non-storable; that is, at any instant in time, the amount of electricity drawn from the grid (demanded) and the amount generated (supplied) should balance. Daily peak load forecasts are extremely important in managing electricity generation and in planning purchases and sales of electricity across utilities (Engle et al., 1992). Moreover, daily peak load forecasts are an important tool for dispatching centers of a power system to schedule maintenance and for adequacy assessment (Amjady, 2001).

From an operational point of view, the key question is whether there will be any problems in meeting peak demand, as failure to meet peak demand can result in power blackouts. The consequences of underpredicting the peak demand go beyond the additional costs incurred by having insufficient capacity to the potentially serious problems of meeting load demand and blackouts. Power blackouts is a critical threat, disturbing the smooth running of the economy and weakening business reliability by restricting business operations in all sectors. To avoid power blackouts, a model that is capable of avoiding the underprediction of the peak load demand, that is, competent in forecasting the upper bound of the peak load demand, is required.

In this paper quantile regression is utilized to construct estimates of the daily peak load demand. Then, the 1.00 quantile (which represents the upper bound of the peak load demand) is forecasted by assuming triangular distribution of the upper tail of the error term. The proposed method is evaluated based on its ability to avoid underprediction.

The coming section discusses the methods that are used in the electricity forecasting literature. Section 3 explains the method proposed and utilized in this study. Section 4, evaluates the forecasting performances of the proposed method, using data from Tokyo Electric Power Company Holdings, Inc. (TEPCO). Section 5, compares and discusses the implications of the results and provides concluding remarks.

2. LITERATURE REVIEW

Over time, different forecasting techniques have been developed to model electricity loads, such as multiple linear regression, the Box–Jenkins approach, and Artificial Neutral Networks (ANN). Most of the previous studies focus on point forecasting Dash et al. (1995); Sadownik and Barbosa (1999); Amjady (2001); Soares and Medeiros (2008); Ohtsuka et al. (2010)). An overview of common methods used in the literature is provided by Weron (2006), and Taylor and McSharry (2007). In addition, Hong (2010) provides a comprehensive review of the load demand modeling and forecasting literature.

Previous studies on peak load demand forecasting mostly focus on forecasting the expected value of the peak load demand (Amjady, 2001; Engle et al., 1992; Ismail et al., 2009; Rallapalli and Ghosh, 2012). However, none of them - to our knowledge - attempt to forecast the upper bound of the peak load demand. In addition, none of them evaluate their proposed models based on their ability to avoid underpredicting the peak load demand.

Quantile regression has not received much attention from the load forecasting community over the past 30 years (Hong and Fan, 2016). In addition, it has rarely been applied in the area of probabilistic energy forecasting (Juban et al., 2016)[1] From the few studies that applied quantile regression in load forecasting, Gibbons and Faruqui (2014), they develop optimal forecast quantile regression method (OFQR) to forecast annual peak load demand. OFQR establishes a loss function framework that uses only annual peak days to estimate the optimal quantile for the model, whereas all days are used to estimate the coefficients of the regression itself. Hong et al. (2014) propose a methodology for computing interval forecasts of electricity demand by applying a quantile regression averaging technique (QRA) to a set of independent expert point forecasts. QRA is a forecast combination approach used to compute prediction intervals. It involves applying quantile regression to the point forecasts of a small number of individual forecasting models or experts. It assigns weights to individual forecasting methods and combines them to yield forecasts of chosen quantiles. Liu et al. (2015) generate probabilistic load forecasts by performing QRA on a set of sister point forecasts. Sister forecasts are predictions generated from the same family of models.

In the current study, quantile regression is employed to construct forecasts of the daily peak demand; then, daily peak demand forecasts are approximated by triangular distribution to generate the upper bound of the daily peak demand.

1 Probabilistic load forecasting provides additional information on the variability and uncertainty of future load values and can be in the form of quantiles, intervals, or density functions (Hong and Fan, 2016).

3. METHOD AND MODELS

Koenker and Bassett (1978) introduce quantile regression model (hereafter QRM) that models conditional quantiles as functions of predictors. Linear regression model specifies the change in the conditional mean of the dependent variable associated with a change in the covariates, whereas the QRM specifies changes in the conditional quantile of the dependent variable associated with a change in the covariates (Hao and Naiman, 2007).

One of the main advantages of quantile regression is that the shape of the distribution does not have to be specified and that any information about these distributions can easily be included in the models (Bremnes, 2004). In other words, quantile regression relaxes linear regression assumptions, and thus it produces flexible, nonsensitive estimates properties that are not found in the linear regression models.

Although it is possible to generate equivalent forecasts of a specific quantile point (which is generated using quantile regression) by constructing linear regression prediction intervals, constructing forecasts using quantile regression is computationally simpler than constructing linear regression prediction intervals. This is because quantile regression is simply a point prediction made using the estimated coefficients and the values of the explanatory variables, without the need for calculating standard errors of the prediction.

Following Koenker and Bassett (1978), the QRM can be expressed as:

$$y_i = \beta_0^{(P)} + \beta_1^{(p)} x_i + \varepsilon_i^{(p)},$$

Where p is the estimated p^{th} quantile, and $0 < p < 1$ indicates the proportion of the population having scores below the quantile p. The QRM minimizes a sum that gives asymmetric penalties, $(1-p)\,|\,y_i - \hat{y}_i\,|$ for overprediction and $p\,|\,y_i - \hat{y}_i\,|$ for under prediction. The p^{th} quantile regression estimates $\beta_0^{(p)}$ and $\beta_1^{(p)}$ are chosen to minimize:

$$\sum_{i=1}^{n} dp(y_i, \hat{y}_i) = p \sum_{y_i \geq \beta_0^{(p)} + \beta_1^{(p)} x_i} |\, y_i - \beta_0^{(p)} - \beta_1^{(p)} x_i \,| + (1-p) \sum_{y_i \leq \beta_0^{(p)} + \beta_1^{(p)} x_i} |\, y_i - \beta_0^{(p)} - \beta_1^{(p)} x_i \,| \tag{1}$$

Where dp is the average weighted distance between y_i and $\hat{y}_i$ As a result, for example if $p = 0.95$, there is a higher penalty for underprediction ($0.95|y_i - \hat{y}_i|$), and a much lower penalty ($0.05|y_i - \hat{y}_i|$) for overprediction. Therefore, the QRM will minimize the positive residuals that are caused by underprediction; accordingly - if applied to peak load demand data - it will minimize the possibility of power blackouts that results from underpredicting the peak load demand.

The QRM can estimate 0.99 or higher quantiles when the sample size is relatively large. However, this method cannot estimate the 1.00 quantile because of the property of its objective function (see Equation 1). In this study the 1.00 quantile point is assumed to be an estimate of the upper bound of the electricity peak demand.

This study introduces the assumption that the distribution of the upper tail of the error term can be approximated by triangular distribution.[2] As Figure 1 indicates, the 1.00 quantile can be estimated from a pair of two quantile points by applying area of triangle rule (for example from 0.99 to 0.98 quantile points or 0.99 and 0.97 quantile points). In the remainder of the paper, the estimated quantile *P* is referred to by $\widehat{y^{(P)}}$.

Since it is possible to model any predetermined position of the distribution, any pair of quantile points can be used to generate the 1.00 quantile point. However, quantile points that are consistent with the needs and aims of this study are chosen. Those are: 0.99, 0.98, 0.97, 0.95 and 0.90 quantile points[3]. For example, using the deviation between the 0.99 and 0.98 quantile points and assuming the triangular distribution, we can calculate the 1.00 quantile point as follows:

$$\widehat{y^{(1.00)}}_{99\&98} = \widehat{y^{(0.99)}} + \left(1+\sqrt{2}\right)*\left(\widehat{y^{(0.99)}} - \widehat{y^{(0.98)}}\right).$$[4]

This relation is easy to obtain by applying the rule for determining the area of the right triangle. When we utilize the deviation between the 0.99 and 0.97 quantile points, the 1.00 quantile point is given by:

$$\widehat{y^{(1.00)}}_{99\&97} = \widehat{y^{(0.99)}} + \left(1+\sqrt{3}\right)*\left(\widehat{y^{(0.99)}} - \widehat{y^{(0.97)}}\right)\big/2.$$

and when the deviation between the 0.99 and 0.95 quantile points is used, the 1.00 quantile point is given by:

$$\widehat{y^{(1.00)}}_{99\&95} = \widehat{y^{(0.99)}} + \left(1+\sqrt{5}\right)*\left(\widehat{y^{(0.99)}} - \widehat{y^{(0.95)}}\right)\big/4.$$

From the deviation between the 0.95 and the 0.90 quantile points, the 1.00 quantile point is given by:

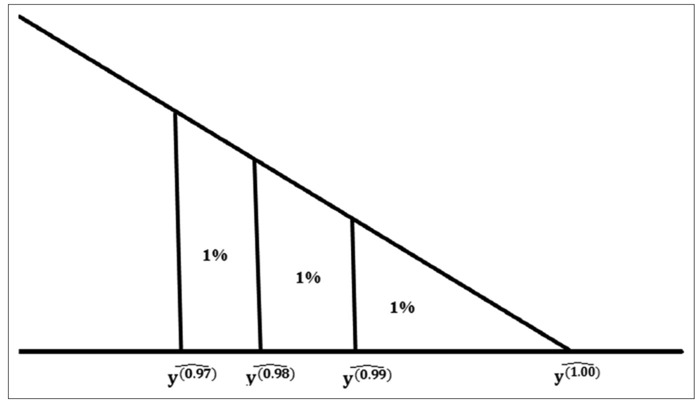
$$\widehat{y^{(1.00)}}_{95\&95} = \widehat{y^{(95)}} + \left(1+\sqrt{2}\right)*\left(\widehat{y^{(0.95)}} - \widehat{y^{(0.90)}}\right).$$

We consider that the 1.00 quantile point is an estimate of the upper limit of the electricity peak demand.

4. EMPIRICAL COMPARISON

To check the performance of the proposed method, it is applied to actual electricity demand data. Realized electricity supply data at every hour from January 1, 2008 to December 31, 2015 is obtained from TEPCO website[5]. The highest electricity supply 10,000 kilowatts between 0:00 and 23:00 h is utilized as the daily peak demand. The reason why we use only the data from January 1, 2012 is used in the analysis is as follows: On March 11, 2011, because of the East Japan Great Earthquake and Tsunami, some nuclear power plants (including the Fukushima first and second power plants) were shutdown. In addition, to save electricity, planned blackouts took place in some areas. Moreover, TEPCO requested its customers to conserve their electricity consumption. The earthquake and its consequences affected the electricity demand pattern and magnitude. Thus it is claimed that the period which is affected by the Fukushima disaster is not suitable for constructing a forecasting model; therefore only data set from January 1, 2012 is utilized.

Figure 1: Quantile points and approximation with triangular distribution



Models are estimated using daily peak demand data from 2012 through 2014, and evaluated the performances of 1.00 quantile points for the in-sample period (2012–2014) and an out-of-sample period (2015).

Figure 2 plots the daily peak demand from 2012 to 2015. It shows strong seasonal fluctuations, with daily peak demand being higher in summer than in winter. The highest yearly peak is observed during August, whereas the lowest peak appears during May. In addition, weekends show a lower peak demand than weekdays.

As an explanatory variable, we adopt the previous day's maximum and minimum temperatures (TempMax and TempMin) and their squared variable (TempMaxSQ and TempMinSQ) at Tokyo because the peak load demand increases when the temperature is low or high[6]. Of course, TEPCO supplies the electricity across the Kanto area, which includes Tokyo metropolitan area. It is hard to calculate the average temperature over Kanto area, so we adopt the temperatures at Tokyo as a representative temperatures. National holidays have a load pattern that differs from working days. In Japan, some of the public holidays' dates change every

2 There might be several candidates to approximate the upper tail distribution that has upper limit, e.g., uniform or Beta distribution. In this paper, we adopt triangular distribution because we can calculate the upper limit (1.00 quantile point) easily and its density is gradually decreases to zero.

3 The aim of this study is to generate a method that prevents the underprediction of peak load demand, thus quantile points that give a high penalty for underprediction are chosen.

4 Details in calculation are described in Appendix.

5 Tokyo Electric Power Company, load demand data. Retrieved on June 10, 2015 from http://www.tepco.co.jp/forecast/html/download-j.html.

6 Daily maximum and minimum temperatures in Tokyo from Japan Meteorological Agency. Retrieved on June 10, 2015 from http://www.data.jma.go.jp/gmd/risk/obsdl/index.php.

Figure 2: Daily peak demand from 2012 to 2015 (in 10,000 kilowatts)

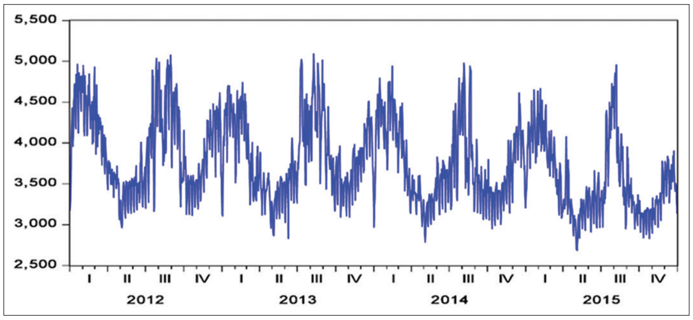


Table 1: Japan's national holidays

Holiday dates	
January 1	Third Monday of July
Second Monday of January	August 11[1)]
February 11	Third Monday of September
March 20 or 21	September 22 or 23
April 29	Second Monday of October
May 3	November 3
May 4	November 23
May 5	December 23

[1)] This holiday commenced in 2016

year, based on the Happy Monday System, which refers to a set of modifications to Japanese laws in 1998 and 2001 to move a number of public holidays to Mondays, creating 3-day weekends for those with 5-day work weeks. Table 1 presents the national holidays of Japan.

In this section, models are estimated using daily peak electricity demand from January 1, 2012 to December 31, 2014 (the in-sample period). Static forecasts are constructed by substituting realized values of the previous period's peak demand into the explanatory variable in the estimated regression, and are then generated for the period January 1, 2015 to December 31, 2015 (the out-of-sample period).

In regard to the dependent variable, logarithms of the series are modeled in order to reduce the effect of heteroskedasticity that may be present because of the characteristics of the data set. First, daily peak demand is expressed as a function of the previous day's peak demand (Log ($Peak_{t-1}$)), and sets of fixed effects for weekends are estimated using the linear regression model utilizing ordinary least squares (OLS). Secondly, we add the previous day's maximum and minimum temperatures (TempMax and TempMin) and their squared variable (TempMaxSQ and TempMinSQ) because the peak load demand increases when the temperature is low or high. Thirdly, we add some dummy variables to account for the month-of-year effect: Dummy variables for the day of the week for Saturday (Sat), Sunday (Sun), Monday (Mon) and dummy variables related with Happy Mondays (HMonday) and Tuesdays after Happy Mondays (HTuesday), and national holiday dummy variables (Holiday). The following models are estimated by OLS.

In Table 2, the OLS estimation results are reported. In Model 1, the estimated coefficient for HMonday is not statistically significant, so we remove it from the equation (Model 2). In the results for the 0.99 quantile of the QRM in Table 2, the estimated coefficients for HMonday, TempMin and TempMinSQ and Holiday are not statistically significant. Then we remove these variables from the equation step by step (Model 3, Model 4 and Model 5) and select Model 5 as the forecasting model. Therefore, in the rest of the paper, we estimate the following model using the QRM:

$$\mathrm{Log}(Peak_t)=\beta_0+\beta_1\log(Peak_{t-1})+\beta_2 TempMax+\beta_3 TempMaxSQ+\beta_6 Sat+\beta_7 Sun+\beta_8 Mon+\beta_{10} HTuesday.$$

In Table 3, the estimation results for the 0.98, 0.97, 0.95, and 0.90 quantile models are reported. Before proceeding to the evaluation of the proposed method, we need to check the performance of the simple forecasting by the QRM. In Table 4, we count the underestimated cases, in which the each quantile points fail to estimate the upper bound and calculate their percentages.

Because the forecasts by the QRM are simply quantile points below one, underestimated cases occurred around the quantiles rates. In other words, we cannot avoid shutdowns owing to shortages in meeting demand. To avoid shutdowns, we must construct the 1.00 quantile point.

In Table 5, we report the numbers and percentages for the underestimated cases for construction of the 1.00 quantile point, approximated by triangular distribution for the in-sample and out-of-sample periods.

Except for the forecasting for the year 2014 and the 1.00 quantiles constructed from the 0.95 to 0.90 pairs, underestimated cases are <0.1% for the in-sample period and are zero for the out-of-sample period in all approximations. From these results, we conclude that we have successfully estimated the upper limits of the daily peak demand. However, if this method is applied for actual peak demand forecasting, we should check the efficiencies of the proposed method. In Table 6, we calculate the average rates of overestimation for the in-sample and out-of-sample periods.

As for the overestimation rates of the forecasting, the 1.00 quantiles constructed from the 0.95 to 0.90 pairs result in a 15.40% overestimation compared with the actual demand, but their performance in terms of underestimation cases is relatively poor. (Table 6) The 1.00 quantiles constructed from the 0.99 to 0.98, 0.99 and 0.97, and 0.99 and 0.95 pairs perform similarly, but the 1.00 quantile estimated from the 0.99 to 0.97 pair results in an overestimate of 16.94% compared with the actual demand, and its rate is smallest compared to other pairs. At this stage, we can conclude that constructing the 1.00 quantile from the 0.99 to 0.97 quantiles is sufficient to estimate the upper limit for the daily peak demand. Of course, this 16.94% overestimation should be considered from other points of view, including its economic and financial consequences. In addition, we should consider another candidate for the independent variables to improve the forecasting models. Furthermore, performances of the 1.00 quantiles from the 0.99 to 0.98 quantiles and from the 0.99 to 0.95 are similar to those from the 0.99 to 0.97 quantiles, so we should compare their performances in other empirical examples.

Table 2: Estimation results for the model selection

	OLS				Quantile regression (0.99 quantile)					
	Model 1		Model 2		Model 3		Model 4		Model 5	
	Coefficients	t-value	Coefficients	t-value	Co4efficients	t-value	Coefficients	t-value	Coefficients	t-value
Constant	0.930077	4.02	0.9424558	4.08	1.973814	2.21	1.44588	2.10	1.480988	2.04
Log ($Peak_{t-1}$)	0.8842679	33.33	0.882844	33.33	0.7938743	7.75	0.84723	10.87	0.8500994	10.19
TempMax	0.0064862	3.17	0.0064085	3.13	−0.0159091	−1.13	−0.0151612	−2.40	−0.01497	−2.25
TempMaxSQ	−0.0001544	−3.15	−0.0001592	−3.12	0.0004106	1.51	0.0003685	2.42	0.0003658	2.31
TempMin	−0.0069036	−4.77	−0.0068996	−4.76	−0.0040892	−0.39	-	-	-	-
TempMinSQ	0.000245	−4.72	0.000245	−4.72	0.000109	0.42	-	-	-	-
Sat	−0.0750605	−17.96	−0.0749538	−17.94	−0.0617837	−2.57	−0.0603984	−3.37	−0.0605366	−3.14
Sun	−0.0384332	−7.96	−0.038709	−8.03	−0.0692341	−3.13	−0.0512656	−2.75	−0.0503069	−2.49
Mon	0.0977679	18.01	0.0963706	18.45	0.093484	4.27	0.1043942	5.49	0.1038784	7.25
HMonday	−0.0133816	−0.94	-	–						
HTuesday	0.057075	5.91	0.057092	5.91	0.04420907	1.73	0.0493612	2.16	0.0489285	2.10
Holiday	−0.0310015	−2.98	−0.0378654	−5.08	−0.0158107	−0.49	−0.0066421	−0.18	-	-
R^2	0.8708		0.8708		0.5098		0.5094		0.5087	
S.E. of regression	0.04641		0.0464		-		-		-	

The R^2 for the OLS is the adjusted R^2, whereas for the quantile regressions, it is a pseudo R^2. OLS: Ordinary least squares

Table 3: Results of other quantile regressions

	0.98 quantile		0.97 quantile		0.95 quantile		0.90 quantile	
	Coefficients	t-value	Coefficients	t-value	Coefficients	t-value	Coefficients	t-value
Constant	1.316022	2.54	1.606325	2.83	1.150462	1.77	0.849201	1.65
Log ($Peak_{t-1}$)	0.865027	14.47	0.830166	12.85	0.882712	12.03	0.912254	16.71
TempMax	−0.01231	−2.53	−0.01272	−2.30	−0.01196	−2.06	−0.00831	−1.47
TempMaxSQ	0.000312	2.69	0.0003	2.29	0.000277	2.01	0.000199	1.51
Sat	−0.05658	−2.69	−0.06199	−3.65	−0.05824	−3.76	−0.06801	−7.01
Sun	−0.05271	−3.38	−0.04971	−4.14	−0.04034	−3.57	−0.03796	−4.22
Mon	0.114612	5.49	0.119097	5.70	0.104304	6.59	0.10494	8.79
HTuesday	0.059603	2.29	0.076297	2.81	0.07442	2.42	0.060164	2.18
R^2	0.5353		0.5553		0.5851		0.6257	

In each year, data from April 1 to March 31 are used for estimations or forecasting. The R^2 for the quantile regressions is a pseudo R^2

Table 4: Underestimated forecasts with 0.95, 0.97, 0.98, and 0.99 quantile regressions

Quantiles	In-sample		Out-of-sample	
	#	%	#	%
0.99	15	1.370	2	0.547
0.98	18	1.644	6	1.644
0.97	28	2.557	9	2.466
0.95	52	4.749	24	6.575

In each year, data from April 1 to March 31 are used for estimation or forecasting. The total number of observations for the in-sample period is 1,095

5. CONCLUSION

Assuming that, in practice, the most crucial issue is to prevent system failure and to eliminate power blackouts, this article aimed to generate a model that is capable of precluding underprediction. 1.00 quantiles constructed by the quantiles estimated by the QRM were compared with the actual demand to investigate the proposed method based on its ability to avoid power blackouts (i.e., to avoid underprediction of demand). From the empirical comparison, we can conclude that we successfully constructed the upper limits of the daily peak demand. From the performance in relation to the overestimation rates, at this stage, we can state that constructing the 1.00 quantile from the 0.99 to 0.97 quantiles is sufficient to estimate the upper limit for the daily peak demand.

Three problems remain to be solved: First, whether the proposed method is useful from an economic or financial perspective; second, which combination of estimated quantiles is best to construct the 1.00 quantile third whether we should consider further candidates for the independent variables to improve the forecasting models. To solve these problems, we need to compare their performances with other empirical data.

Appendix: Calculation of 100% quantile.

From Figure 1, we set a and b as follows:

$a = \overline{v^{(0.99)} - v^{(0.98)}}$ and $b = \widehat{y^{(1.00)}} - \widehat{y^{(0.99)}}$.

Formula for triangular area is:

$$\text{Area} = \frac{1}{2}\left(\text{base} * \text{ height}\right).$$

and tan θ and height are defined as:

tan θ=height/base and height=tan θ * base.

Then we can rewrite the formula to:

Table 5: Underestimation by forecasting with triangular approximation

Year	In or out of sample	0.99 and 0.98		0.99 and 0.97		0.99 and 0.95		0.95 and 0.90	
		#	%	#	%	#	%	#	%
2012	In-sample	0	0.00	0	0.00	0	0.00	0	0.00
2013	In-sample	1	0.09	0	0.00	0	0.00	5	1.37
2014	In-sample	3	0.82	2	0.54	1	0.09	5	1.37
From 2013 to 2014		4	0.36	2	0.18	1	0.09	10	0.91
2015	Out-of-sample	0	0.00	0	0.00	0	0.00	1	0.27

#and % indicate the number and percentage of underestimations, respectively. Bold faced numbers are best performed in each year forecasts

Table 6: Ratios of overestimation to actual electricity demand

Year	In or out of sample	0.99 and 0.98%	0.99 and 0.97%	0.99 and 0.95%	0.95 and 0.90%
2012	In-sample	15.72	16.15	16.27	14.08
2013	In-sample	15.78	16.23	16.52	14.15
2014	In-sample	16.21	16.37	16.86	14.54
From 2013 to 2014		18.41	15.90	16.25	16.55
2015	Out-of-sample	17.09	16.94	17.76	15.40

The symbol % indicates the percentage of overestimations compared with actual electricity demand

$$\text{Area} = \frac{1}{2} * base^2 \tan\theta \; .$$

Figure 1 means:

$$\frac{1}{2} * b^2 \tan\theta = 1\% \text{ and } \frac{1}{2} * (a+b)^2 \tan\theta = 2\%,$$

Then the following equality is obtained:

$$\frac{(a+b)^2 \tan\theta}{2} = b^2 \tan\theta,$$

and remove tan θ from both sides:

$$\frac{1}{2}(a+b)^2 = b^2 \; .$$

Solution of quadratic function is given as:

$$\text{b} = \frac{2a \pm \sqrt{4a^2 + 4a^2}}{2},$$

We should take positive solution, so

$$\text{b} = a(1+\sqrt{2})$$

Then we can calculate 1.00 quantile point as

$$\widehat{\text{y}^{(1.00)}}_{99\&98} = \widehat{\text{y}^{(0.99)}} + \left(1+\sqrt{2}\right) * \left(\widehat{\text{y}^{(0.99)}} - \widehat{\text{y}^{(0.98)}}\right).$$

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