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Vector Autoregressive with Exogenous Variable Model and its Application in Modeling and Forecasting Energy Data: Case Study of PTBA and HRUM Energy

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ABSTRACT

Owing to its simplicity and less restrictions, the vector autoregressive with exogenous variable (VARX) model is one of the statistical analyses frequently used in many studies involving time series data, such as finance, economics, and business. The VARX model can explain the dynamic behavior of the relationship between endogenous and exogenous variables or of that between endogenous variables only. It can also explain the impact of a variable or a set of variables on others through the impulse response function (IRF). Furthermore, VARX can be used to predict and forecast time series data. In this study, PTBA and HRUM energy as endogenous variables and exchange rate as an exogenous variable were studied. The data used herein were collected from January 2014 to October 2017. The dynamic behavior of the data was also studied through IRF and Granger causality analyses. The forecasting data for the next 1 month was also investigated. On the basis of the data provided by these different models, it was found that VARX (3,0) is the best model to assess the relationship between the variables considered in this work.

Keywords: Vector Autoregressive Model, Vector Autoregressive with Exogenous Variable Model, Granger Causality, Impulse Response Function, Forecasting
JEL Classifications: C32, Q4, Q47

1. INTRODUCTION

Currently, the development of communication technology and the globalization of the economy have accelerated the integration of world financial markets. The aim of empirical economic analysis is to investigate the economics dynamic and its mechanisms. To this end, relevant economic data are needed (Gourieroux and Monfort, 1997). Price movement in one market can easily spread to other markets. Therefore, financial markets are more dependent on each other and must be considered jointly to better understand the dynamics of global finance (Tsay, 2005; 2014). The vector autoregressive (VAR) model plays an important role in modern techniques of analysis, especially in economics and finance (Hamilton, 1994; Kirchgassner and Wolters, 2007). The VAR model was introduced by Sims (1980) as a method to analyze macroeconomic data. He developed the VAR model as an alternative to the traditional system, which involved several equations (Kirchgassner and Wolters, 2007). VAR is one of the most used research tools to analyze macroeconomic time series data in the last two decades. There are some advantages because of which the VAR model is commonly used to analyze multivariate time series: (1) The model is relatively easy to estimate, i.e., for a VAR model, LSE are asymptotically equivalent to the method of MLE and OLS; (2) the properties of the VAR model have been extensively discussed in the literature; (3) the VAR model is similar to multivariate multiple linear regression (Tsay, 2014). Sims stressed the need to drop the adhoc dynamic restrictions in regression models and to discard empirically implausible exogeneity assumptions (Sims, 1980). He also stressed the need to jointly model all endogenous variables rather than one equation at a time (Kilian, 2011).

The VAR model is often used to describe the behavior of a variable over time (Al-hajj et al., 2017; Sharma et al., 2018). In this model, it is assumed that the current value can be expressed as a function of preceding values and a random error (Fuller, 1985). Hence, VAR is an easy model to analyze multivariate time series data; the VAR model is also flexible, easy to estimate, and usually gives a good fit to the data (Juselius, 2006; Fuller, 1985, Tsay, 2014, Lutkepohl, 2005). The VAR model, which involves a normal distribution, has frequently been a popular choice as a description of macroeconomic time series data (Juselius, 2006). In a VAR model of order p, VAR (p), each component of vector X_t depends linearly on its own lagged values up to p periods as well as on the lagged values of all other variables up to lag p (Wei, 1990; Lutkepohl, 2005; Tsay, 2005 and 2014; Kirchgassner and Wolters, 2007). The VAR model is extremely useful for describing and explaining the behavior of financial, business, and economic time series data and also for forecasting (Wei, 1990; Lutkepohl, 2005; Al-hajj et al., 2017; Sharma et al., 2018). Forecasting is the primary objective in the analysis of multivariate time series data. Forecasting using the VAR model is simple because it can be conditioned by potential future paths of specified variables in the model. Furthermore, the VAR model can be used for structural analysis. In structural analysis, certain assumptions are imposed on the causal structure of the data under investigation, and the resultant causal impacts of unexpected shocks or innovations to specified variables are studied. These causal impacts are usually summarized in Granger causality and impulse response function (IRF) (Wei, 1990; Hamilton, 1994; Lutkepohl, 2005). As our study involves independent or exogenous variables, the VAR model can be easily extended to a VAR model with exogenous variable and referred to as the VAR with exogenous variable (VARX) model (Hamilton, 1994; Tsay, 2015). The VARX model is also called a dynamic model (Gourieroux and Monfort, 1997).

2. STATISTICAL MODEL

The assumption of the stationary state in time series data analysis is fundamental and must be checked before analyzing the data. Some methods are available to check the stationary state of the time series data based on data plots or through the augmented Dickey Fuller test (ADF test). The process of the ADF test is as follows (Brockwell and Davis, 2002; Tsay, 2005). Let $x_1, x_2., x_n$ be the time series, and assume that $\{x_t\}$ follows the AR(p) model with mean μ given by:

$$x_t - \mu = \phi_1 (x_{t-1} - \mu) + \ldots + \phi_p (x_{t-p} - \mu) + \varepsilon_t \tag{1}$$

Where ε_t is white noise with mean 0 and variance σ^2, and $\varepsilon_t \sim WN(0, \sigma^2)$. The model (1) can be written as:

$$\nabla x_t = \phi_0^* + \phi_1^* x_{t-1} + \phi_2^* \nabla x_{t-1} + \ldots + \phi_p^* \nabla x_{t-p+1} + \varepsilon_t \tag{2}$$

Here,

$$\phi_0^* = \mu(1 - \phi_1 - \ldots - \phi_p),$$

$$\phi_1^* = \sum_{i=1}^{p} \phi_i - 1,$$

$$\phi_j^* = \sum_{i=j}^{p} \phi_i,$$

j = 2,3., p,

And $\nabla x_t = x_t - x_{t-1}$

The testing of the nonstationarity data of model (2) using the ADF or tau (τ) tests is conducted as follows. Ho: $\phi_1^* = 0$ (nonstationary data) against Ha: $\phi_1^* < 0$ (stationary data). The statistics test is (ADF test)

$$\text{ADF test}(\tau) = \frac{\hat{\phi}_1^*}{\hat{S}e_{\phi_1^*}} \tag{3}$$

For the level of significance ($\alpha = 0.05$), reject Ho if $\tau < -2.57$ or if the $P < 0.05$ (Brockwell and Davis, 2002; Tsay, 2005).

Time series data in economics, finance, business, or social sciences are collected at equal time intervals, such as days, weeks, months, quarters, or years. In many cases, such time series data may have related variables of interest. Hence, to know a variable better, it must be explained by other variable(s). Therefore, the variables must be analyzed jointly (Wei, 1990; Hamilton, 1994; Lutkepohl, 2005; Pena et al. 2001). The reasons why the model presents these time series together (Pena et al. 2001) are (1) to understand the dynamic relationship between the time series and (2) to improve the forecast's accuracy. Apart from these reasons, the structure of the relationship between the time series data could also be of interest. Maybe, there are hidden factors responsible for the dynamic improvement of time series data. Let $\{x_{1t}\}, \{x_{2t}\}.,\{x_{kt}\}$ t = 0, ±1, ±2., k time series data taken at equal time intervals, and $\Gamma_t = \{x_{1t}, x_{2t}., x_{kt}\}$, where Γ_t is also called a k-dimensional vector time series (VTS). The analysis of VTS data has been extensively discussed in the literature (Wei, 1990; Lutkepohl, 2005; Tsay, 2005). If the mean of $E(x_{it}) = \mu_i$ is constant for each i = 1, 2., k and the cross covariance between x_{it} and x_{js} for all i = 1, 2., k and j = 1, 2., k is a function of only the time difference ts. Therefore, the equation is.

$$E(\Gamma_t) = \mu = \begin{bmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_k \end{bmatrix} \tag{4}$$

And the covariance matrix is:

$$\sum (k) = \text{Cov}(\Gamma_t, \Gamma_{t+m}) = E[(\Gamma_t - \mu)(\Gamma_{t+m} - \mu)']$$

$$= \begin{bmatrix} \gamma_{11}(m) & \gamma_{12}(m) & \ldots & \gamma_{1k}(m) \\ \gamma_{21}(m) & \gamma_{22}(m) & \ldots & \gamma_{2k}(m) \\ \vdots & \vdots & \vdots & \vdots \\ \lambda_{k1}(m) & \gamma_{k2}(m) & \ldots & \gamma_{kk}(m) \end{bmatrix} = \text{Cov}(\Gamma_{t+m,} \Gamma_t) \tag{5}$$

Where,

$\gamma_{ij}(m) = E(X_{it} - \mu_i)(X_{i,t+m} - \mu_j) = E(X_{i,t-m} - \mu_i)(X_{j,t} - \mu_j)$

2.1. VAR (p) and VARX (p,q) Models

The general VAR (p) model is as follows:

$$\Gamma_t = \Phi_1 \Gamma_{t-1} + \Phi_2 \Gamma_{t-2} + \ldots + \Phi_p \Gamma_{t-p} + E_t \quad (6)$$

Or

$$(I - \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p)\, \Gamma_t = E_t \quad (7)$$

Where $B^j \Gamma_t = \Gamma_{t-j}$ and j = 1, 2., p. $\Phi_s = \left[\phi_{lm}^{(s)}\right]$ is matrix k × k and s = 1, 2., p.

2.2. Condition for Stationary

Rewriting VAR (p) as VAR (1)

$$\xi_t = \begin{bmatrix} \Gamma_t - \mu \\ \Gamma_t - \mu \\ \vdots \\ \Gamma_{t-p+1} - \mu \end{bmatrix}_{npx1} \quad F = \begin{bmatrix} \Phi_1 & \Phi_2 & \ldots & \Phi_{p-1} & \Phi_p \\ I_n & 0 & \ldots & 0 & 0 \\ 0 & I_n & \ldots & 0 & 0 \\ \vdots & \vdots & \ldots & \vdots & \vdots \\ 0 & 0 & \ldots & I_n & 0 \end{bmatrix}_{npxnp} \text{ and}$$

$$v_t = \begin{bmatrix} \varepsilon_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}.$$

Then, VAR (p) can be rewritten as VAR (1):

$$\xi_t = F\, \xi_{t-1} + v_t \quad (8)$$

Condition for stationary proposition (Hamilton, 1994)

The Eigen value of matrix F satisfies

$$|I_n \lambda - \Phi_1 \lambda^{p-1} - \Phi_2 \lambda^{p-2} - \ldots - \Phi_p| = 0$$

And it is covariance stationary as long as $|\lambda| < 1$ for all values of λ. Otherwise, equivalently, the VAR is covariance stationary if all values of z satisfy

$$|I_n - \Phi_1 z - \Phi_2 z^2 - \ldots - \Phi_p z^p| = 0$$

The root are lies outside the unit circle.

The correlation matrix for the vector process is as follows.

$$\rho(m) = D^{-½} \Sigma(m) D^{-½} = [\rho_{ij}(m)].$$

I = 1, 2., m; j = 1, 2., m,

$$D = [\mathrm{diag}\, [\gamma_{11}(0), \gamma_{22}(0), \ldots, \gamma_{mm}(0)],$$

and

$$\rho_{ij}(m) = \frac{\gamma_{ij}(m)}{[\gamma_{ii}(0)\ \gamma_{jj}(0)]^{1/2}} \quad (9)$$

Represents the cross-correlation function between x_{it} and x_{jt}.

A basic assumption in model (1) is that the error vector following multivariate white noise is as follows:

$$E(\varepsilon_t) = 0.$$

$$E(\varepsilon_t\, \varepsilon_s') = \begin{cases} \Sigma_\varepsilon & \text{if } t = s \\ 0 & \text{if } t \neq s \end{cases}.$$

A VAR process can be affected by an exogenous variable, which can be stochastic or nonstochastic. The VAR process can also be affected by the lag of the exogenous variables.

The VARX (p,q) model is expressed by the following equation.

$$\Gamma_t = c + \sum_{i=1}^{p} \phi_i \Gamma_{t-i} + \sum_{j=0}^{q} \varphi_j \Psi_{t-j} + \varepsilon_t \quad (10)$$

When p = 3, q = 0, $\Gamma_t = \begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix}$, $\Psi_t = y_t$, $\phi_i = \begin{bmatrix} \phi_{11}^i & \phi_{12}^i \\ \phi_{21}^i & \phi_{22}^i \end{bmatrix}$, where i = 1,2,3, $\varphi_j = \begin{pmatrix} \varphi_{11} \\ \varphi_{21} \end{pmatrix}$, and $c = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$, then the VARX (3,0) becomes.

$$\begin{pmatrix} x_{1t} \\ x_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{bmatrix} \phi_{11}^1 & \phi_{12}^1 \\ \phi_{21}^1 & \phi_{22}^1 \end{bmatrix} \begin{pmatrix} x_{1t-1} \\ x_{2t-1} \end{pmatrix} + \begin{bmatrix} \phi_{11}^2 & \phi_{12}^2 \\ \phi_{21}^2 & \phi_{22}^2 \end{bmatrix} \begin{pmatrix} x_{1t-2} \\ x_{2t-2} \end{pmatrix} + \begin{bmatrix} \phi_{11}^3 & \phi_{12}^3 \\ \phi_{21}^3 & \phi_{22}^3 \end{bmatrix} \begin{pmatrix} x_{1t-3} \\ x_{2t-3} \end{pmatrix} + \begin{pmatrix} \varphi_{11} \\ \varphi_{12} \end{pmatrix} y_t + \varepsilon_t$$

2.3. Economic Test for Granger Causality

Here, we perform econometric tests of whether a particular observed series Y Granger-Causes X can be based on the following model (Hamilton, 1994) to let a particular autoregressive lag length p and estimate.

$$X_t = c_t + \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + \ldots + \beta_p Y_{t-p} + u_t \quad (11)$$

Through OLS assumption, the null hypothesis is H_0: $\beta_1 = \beta_2 = \ldots \beta_p = 0$; hence, Sum squared Residual from model (11) is calculated as

$$RSS_1 = \sum_{i=1}^{T} \hat{u}_t^2$$

Under null hypothesis, the model is

$$X_t = c_0 + \gamma_1 X_{t-1} + \gamma_2 X_{t-2} + \ldots + \gamma_p X_{t-p} + \varepsilon_t \quad (12)$$

To calculate Sum squared residual from model (12), we use

$$RSS_0 = \sum_{i=1}^{T} \hat{\varepsilon}_t^2$$

Finally, the statistics test provides

$$F = \frac{(RSS_o - RSS_1)/p}{RSS_1/(T-2p-1)} \quad (13)$$

Ho is rejected if $F > F_{0.05;\,(p,\, T-2p-1)}$.

2.4. IRF

The VAR model can be written in vector MA (∞) as

$$X_t = \mu_0 + \varepsilon_t + \Psi_1 \varepsilon_{t-1} + \Psi_2 \varepsilon_{t-2} \ldots$$

Thus, the matrix Ψ_s is interpreted as

$$\frac{\partial X_{t+s}}{\partial \varepsilon_t'} = \Psi_s$$

The row i, column j element of Ψ_s identifies the effects of an increase with one unit in the j[th] variable's innovations at date t (ε_{jt}) for the value of the i[th] variable at time t + s ($X_{i,t+s}$), while maintaining all other innovations at constant dates. If the first element of ϵ_t is changed by δ_1, the second element is simultaneously changed by δ_2, and the n[th] element is changed by δ_n; then, the combined effect of these changes on the value of vector X_{t+s} would be

$$\Delta X_{t+s} = \frac{\partial X_{t+s}}{\partial \varepsilon_{1t}} \delta_1 + \frac{\partial X_{t+s}}{\partial \varepsilon_{2t}} \delta_2 + \ldots + \frac{\partial X_{t+s}}{\partial \varepsilon_{nt}} \delta_n = \Psi_s \delta. \quad (14)$$

A plot of the row i, column j element of Ψ_s is

$$\frac{\partial X_{i,t+s}}{\partial \varepsilon_{jt}},$$

Where in a function of s is called IRF.

2.5. Forecasting

Forecasting is one of the main objectives in the analysis of multivariate time series data. Forecasting in a VAR (p) model is basically similar to forecasting in a univariate AR (p) model. First, the basic idea in the process of forecasting is that the best VAR model must be identified using certain criteria for choosing the best model. Once the model is found, it can be used for forecasting. Similarly, the VARX (p,q) model (10) with the parameters ϕ_i for i = 1, 2., p and φ_j for j = 1, 2., q in equation (10) is assumed to be known. The best predictor, in terms of minimum mean squared error, for Γ_{t+1} or 1−step forecast based on the available data at time T is as follows.

$$\Gamma_{T+1|T} = \hat{c} + \hat{\phi}_1 \Gamma_T + \hat{\phi}_2 \Gamma_{T-1} + \ldots + \hat{\phi}_p \Gamma_{T-p+1} + \hat{\phi}_0 \Psi_T + \hat{\phi}_1 \Psi_{T-1} + \ldots + \hat{\phi}_q \Psi_{T-q} \quad (15)$$

Forecasting for longer durations, for example h-step forecast, can be obtained using the chain rule of forecasting as expressed below.

$$\Gamma_{T+h|T} = \hat{c} + \hat{\phi}_1 \Gamma_{T+h-1|T} + \hat{\phi}_2 \Gamma_{T+h-2|T} + \ldots + \hat{\phi}_p \Gamma_{T+h-p|T} + \hat{\phi}_0 \Psi_{T+h|T} + \hat{\phi}_1 \Psi_{T+h-1|T} + \ldots + \hat{\phi}_q \Psi_{T+h-q|T} \quad (16)$$

3. RESULTS AND DISCUSSION

The data used in this study are HRUM energy and PTBA closing price, which were collected from January 2014 to October 2017 (LQ45a, 2018; LQ45b, 2018). The data Exchange Rate are also taken from January 2014 to October 2017 (Bank Indonesia, 2018). The data HRUM energy and PTBA are adopted from LQ45, and the Exchange Rate is adopted from the Bank Indonesia. The plot of these data is given in Figure 1.

From Figure 1, it can be seen that the data for Exchange rate, PTBA, and HRUM energy are nonstationary. The data for PTBA from January 2014 to October 2015 show an increasing trend, those from October 2015 to January 2016 show a decrease; in the data from January 2016 to September 2016 the trend still decreases, and from September 2016 to October 2017 the trend is flat, but with significant fluctuations. The data for HRUM energy from January 2014 to January 2016 shows a decreasing trend; for the data from January 2016 to September 2016 the trend increases, and from September 2016 to October 2017 the trend is flat, but with fluctuations. The data for the Exchange Rate from January 2014 to May 2015 shows an increasing trend, which slowly decreases and fluctuates from May 2015 to May 2016; this trend is finally flat from May 2016 to October 2017. Data analysis conducted using the ADF test shows nonstationary data (Table 1). The next step is to differentiate the data to make them stationary in mean. Figure 2 shows the data obtained after differentiation with d = 1.

By differentiation with d = 1, the HRUM energy and PTBA data become stationary. To find the best model, several models, namely, VARX (1) – VARX (5), were compared and the information criteria AICC, HQC, AIC, and SBC were used. The best fit was correlated with the smallest values of those criteria, which are listed in Table 2.

Based on the values in Table 2, it is found that the best model is VARX (1,0), with the minimum value of SBC of 18.873. According to the HQC criteria, the best model is VARX (3,0), with a minimum value of 18.8516. The AICC and AIC criteria indicated VARX (4,0) as the best model, with minimum values of 18.8140 and 18.8136, respectively. The schematic representation of parameter estimates for the VARX (1,0), VARX (3,0), and VARX (4,0) models are given in Table 3.

According to the data in Table 3, three parameters (AR1) are significant in the VARX (1,0) model, and six parameters (AR1−3) are significant (sign: – and +) in the VARX (3,0) and VARX (4,0) models. Because no parameters are significant in AR4, model VARX (3,0) is used as the best model for the data.

Model VARX (3,0) is expressed by the following equation.

$$\Gamma_t = \begin{pmatrix} 169.505 \\ 22.929 \end{pmatrix} + \begin{pmatrix} 0.962 & -0.039 \\ -0.005 & 1.0704 \end{pmatrix} \Gamma_{t-1} + \begin{pmatrix} -0.039 & -0.681 \\ -0.001 & -0.144 \end{pmatrix} \Gamma_{t-2} + \begin{pmatrix} 0.061 & 0.777 \\ 0.004 & 0.070 \end{pmatrix} \Gamma_{t-3} + \begin{pmatrix} -0.008 \\ -0.001 \end{pmatrix} \Psi_t + \varepsilon_t \quad (17)$$

With

$$\Sigma_\varepsilon = \begin{pmatrix} 72419.29 & 19.23 \\ 19.23 & 2023.68 \end{pmatrix}$$

Where Ψ_t = exchange rate$_t$. Model VARX (3,0) can also be written as two univariate regression models:

Figure 1: Exchange rate, PTBA, and HRUM energy from January 2014 to October 2017

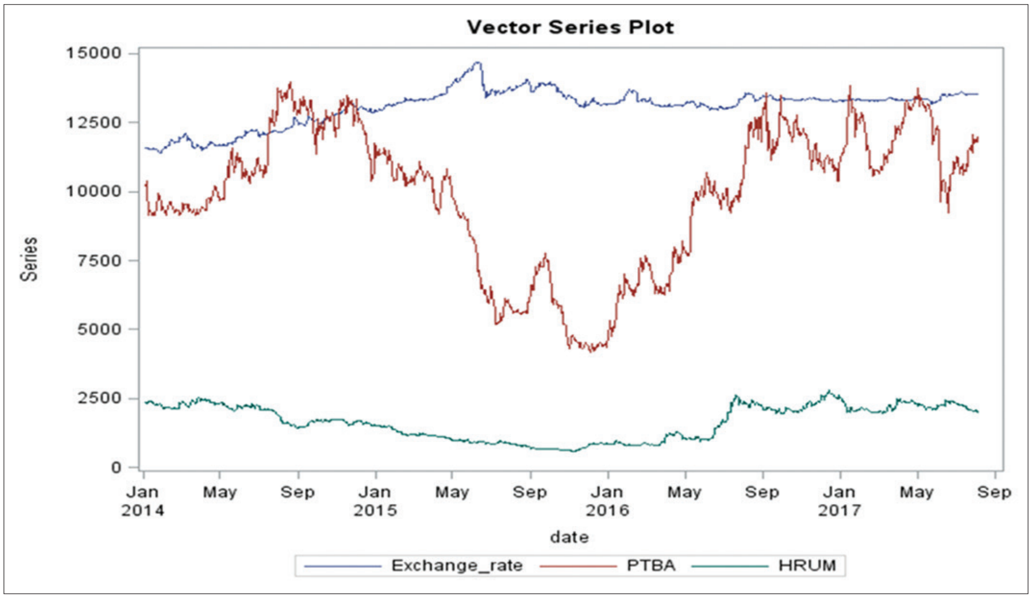


Figure 2: Residual plot, ACF, PACF, and IACF after differentiation with d = 1 for (a) PTBA and (b) HRUM energy data

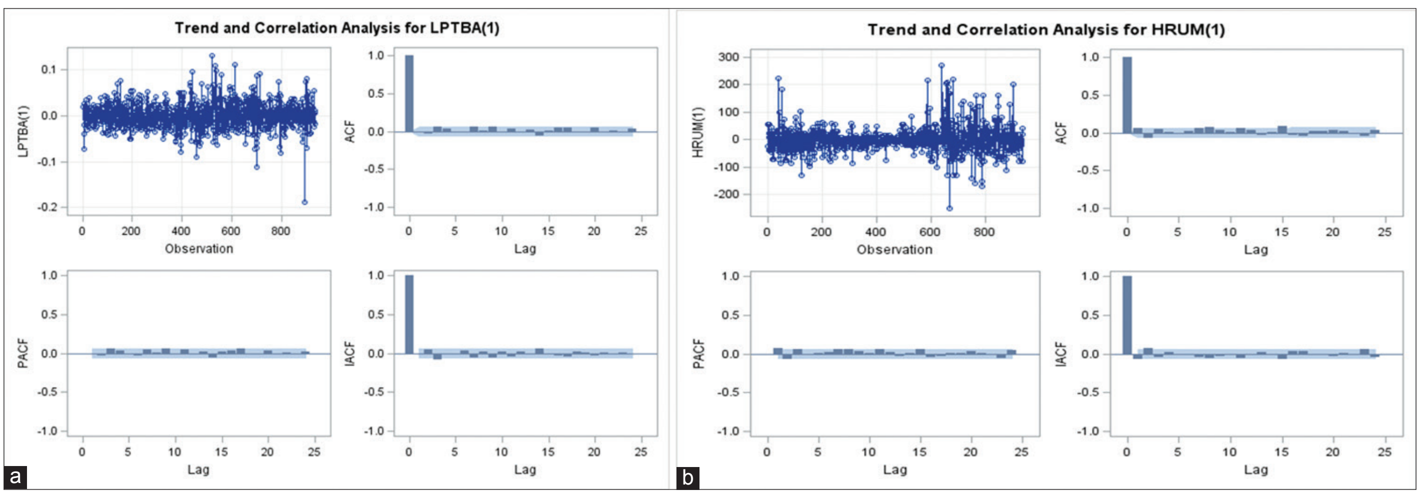


Table 1: ADF test for data PTBA and HRUM Energy before and after differentiation (d=1)

Variable	Type	Before differentiation				After differentiation (d=1)			
		Rho	P value	Tau	P value	Rho	P value	Tau	P value
PTBA	Zero mean	−0.17	0.6448	−0.21	0.6111	−1088.4	0.0001	−23.32	<0.0001
	Single mean	−4.74	0.4605	−1.50	0.5357	−1088.5	0.0001	−23.31	<0.0001
	Trend	−4.89	0.8286	−1.54	0.8159	−1089.3	0.0001	−23.30	<0.0001
HRUM Energy	Zero mean	−0.57	0.5561	−0.67	0.4272	−1003.0	0.0001	−22.37	<0.0001
	Single mean	−3.21	0.6313	−1.35	0.6065	−1003.2	0.0001	−22.36	<0.0001
	Trend	−3.64	0.9068	−1.53	0.8199	−1009.5	0.0001	−22.42	<0.0001

ADF: Augmented Dickey Fuller test

Table 2: Comparison of the criteria for VARX (1,0)–VARX (5,0) models

Information criteria	VARX (1,0)	VARX (2,0)	VARX (3,0)	VARX (4,0)	VARX (5,0)
AICC	18.8360	18.8401	18.8202	18.8140*	18.8187
HQC	18.8517	18.8637	18.8516*	18.8531	18.8656
AIC	18.8359	18.8400	18.8199	18.8136*	18.8181
SBC	18.8773	18.9022	18.9029	18.9174	18.9427

$$\Gamma_{1t} = 169.505 + 0.962\,\Gamma_{1t-1} - 0.039\,\Gamma_{2t-2} - 0.039\,\Gamma_{1t-2} - 0.68\,\Gamma_{2t-2} + 0.061\,\Gamma_{1t-3} + 0.777\,\Gamma_{2t-3} - 0.001\,\Psi_t + \varepsilon_{2t} \quad (18)$$

And

$$\Gamma_{2t} = 22.929 - 0.005\,\Gamma_{1t-1} + 1.070\,\Gamma_{2t-1} - 0.001\,\Gamma_{1t-2} - 0.144\,\Gamma_{2t-2} + 0.0004\,\Gamma_{1t-3} + 0.070\,\Gamma_{2t-3} - 0.008\,\Psi_t + \varepsilon_{1t} \quad (19)$$

The statistical test results of the parameters in model (17) are presented in Table 4, and those for models (18) and (19) are presented in Table 5. The results of the statistical test indicate that model (18) is very significant, with the statistical test $F = 12124.2$ with $P < 0.0001$. The degree of determination, R-squared, is 0.9892. This means that 98.92% of the variation of Γ_{1t} (PTBA) can be explained by lag variables of Γ_{1t-1}, Γ_{2t-1}, Γ_{1t-2}, Γ_{2t-2}, Γ_{1t-3}, Γ_{2t-3} and Ψ_t (Exchange rate). According to the results obtained from the statistical test, model (19) is very significant, with the statistical test $F = 25452.8$ and $P < 0.001$. The degree of determination, R−Squared, is 0.9948. This means that 99.48% of the variation of Γ_{2t} (HRUM Energy) can be explained by the lag variables of Γ_{1t-1}, Γ_{2t-1}, Γ_{1t-2}, Γ_{2t-2}, Γ_{1t-3}, Γ_{2t-3} and Ψ_t (Exchange rate).

Table 3: Schematic representation of parameter estimates for the VARX (1,0), VARX (3,0), and VARX (4,0) models

Model	Variable/lag	C	XL0	AR1	AR2	AR3	AR4
VARX (1,0)	PTBA	•	•	++			
	HRUM	•	•	•+			
VARX (3,0)	PTBA	•	•	+•	•–	•+	
	HRUM	•	•	•+	•–	•+	
VARX (4,0)	PTBA	•	•	+•	•–	+•	••
	HRUM	•	•	•+	•–	•+	••

+ : >2* Standard error, – : <–2* Standard error, •: is between, *: N/A

3.1. IRF

In economics, IRF is used to describe how economics reacts over time to the exogenous impulse, which economists usually call shock and model in the context of VAR. Figure 3 shows the IRF shock in exchange rate. One standard deviation in the exchange rate causes PTBA to respond negatively and increase up to 2 years. The minimum effect occurs in lag 0 (the 1st day) with the value about −0.007 and shifts to zero (stable condition) up to 2 years (about 720 days (Figure 3a). Despite this, the negative impact of PTBA is extremely small yet considerably significant in these horizons until the 2nd year. The shock of one standard deviation in the exchange rate also causes HRUM energy to respond negatively and increase until the 2nd year (about 720 days). The minimum effect occurs in lag 0 (the 1st day) with the value about −0.001 and moves to zero (stable condition) up to 2 years (about 720 days (Figure 3b). Even so, the negative impact on HRUM energy is very small and close to zero yet highly significant in these horizons up to two years. Figure 4a shows the impulse in PTBA. The shock of one standard deviation in PTBA causes PTBA to respond positively and have significance for about 3 months, whereas from the third up to the 7 month, the response moves to zero (stable condition). Thus, the stable condition is reached up to the 7th month. It is interesting to see the behavior of the confidence interval between the third and 14th month when the volatility is high (Figure 4a). The impulse in PTBA seems to have an effect on the volatility of HRUM energy. Figure 4b shows that the HRUM energy is still stable around zero when a shock in PTBA was noticed, but the volatility is high up to one year after the shock in PTBA, which indicates that the closing price of HRUM energy fluctuates within one year following the shock in PTBA.

Figure 5a illustrates the IRF shock in HRUM energy. The shock of one standard deviation in HRUM energy causes PTBA to

Table 4: Statistical test for the parameters used in model (17)

Equation	Parameter	Estimate	Standard error	t value	P value	Variable
PTBA	CONST1	169.505	213.8244	0.79	0.4281	1
	XL0_1_1	−0.008	0.01545	0.53	0.5935	Exchange
	AR1_1_1	0.962−	0.03251	29.61	0.0001	rate (t)
	AR1_1_2	0.039−	0.19602	−0.20	0.8409	PTBA (t-1)
	AR2_1_1	0.039−	0.04519	−0.87	0.3872	HRUM (t-1)
	AR2_1_2	0.681	0.28645	−2.38	0.0176	PTBA (t-2)
	AR3_1_1	0.061	0.03241	1.89	0.0585	HRUM (t-2)
	AR3_1_2	0.777	0.19695	3.95	0.0001	PTBA (t-3)
						HRUM (t-3)
HRUM	CONST2	22.929	35.7438	0.64	0.5214	1
	XL0_2_1	−0.001	0.00258	−0.49	0.6246	Exchange
	AR1_2_1	−0.00357	0.00543	−0.66	0.5116	rate (t)
	AR1_2_2	1.07042	0.03277	32.67	0.0001	PTBA (t-1)
	AR2_2_1	−0.00058	0.00755	−0.08	0.9391	HRUM (t-1)
	AR2_2_2	−0.14383	0.04788	−3.00	0.0027	PTBA (t-2)
	AR3_2_1	0.00400	0.00542	0.74	0.4604	HRUM (t-2)
	AR3_2_2	0.07015	0.03292	2.13	0.0334	PTBA (t-3)
						HRUM (t-3)

Table 5: Univariate diagnostic checks

Model	Variable	R_squared	Standard deviation	F value	P value
(18)	PTBA	0.9892	269.11	12124.2	<0.0001
(19)	HRUM	0.9948	44.985	25452.8	<0.0001

Figure 3: (a and b) Impulse response function in exchange rate

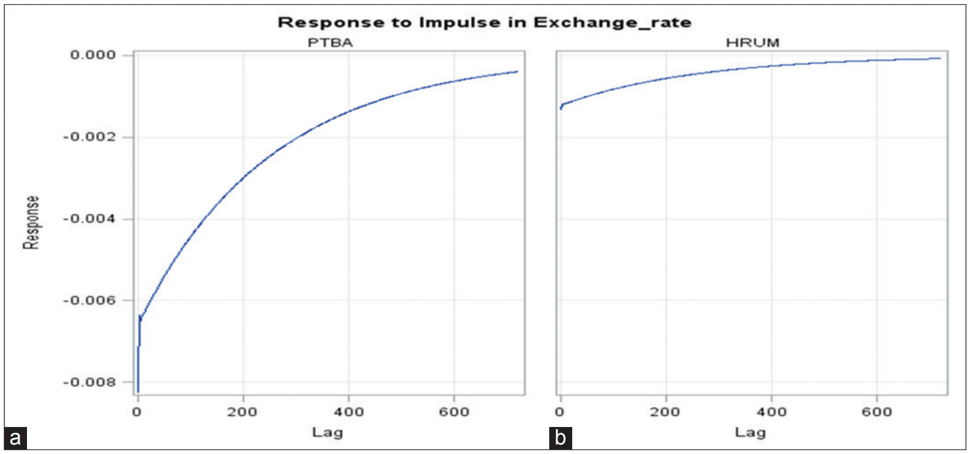


Figure 4: (a and b) Impulse response function in PTBA

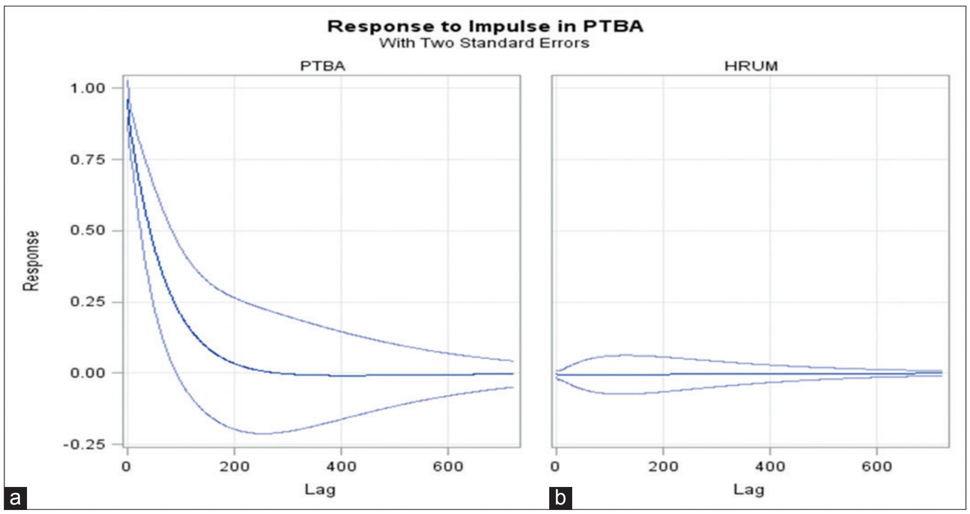
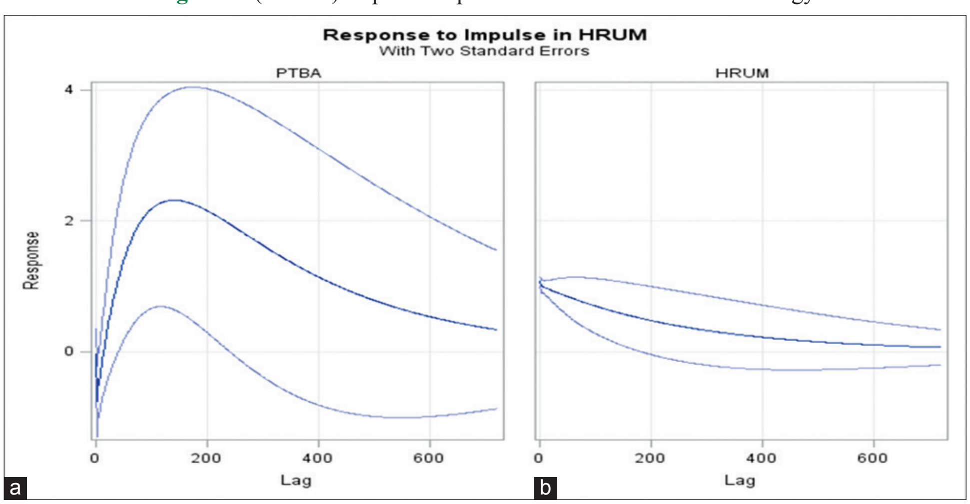


Figure 5: (a and b) Impulse response function IRF in HRUM energy



respond negatively for about 2 weeks; however, after that the impact is positive up to 2 years. The positive impact reaches maximum in about the 5th month, then it decreases to zero (stable condition) after 2 years (about 720 days). The impact is positive and very significant in the range of 2 weeks up to the 7 month because the confidence interval in this range does not include zero (Figure 5a). From the behavior of the confidence interval (Figure 5a), it can be observed that the volatility is very high. Hence, it can be concluded that in this horizon, the closing price after the shock of HRUM energy fluctuates significantly. The impact of impulse in HRUM energy causes HRUM energy to respond positively and tend to zero in 2 years (about 720 days).

Figure 6: Prediction and forecasting of PTBA data

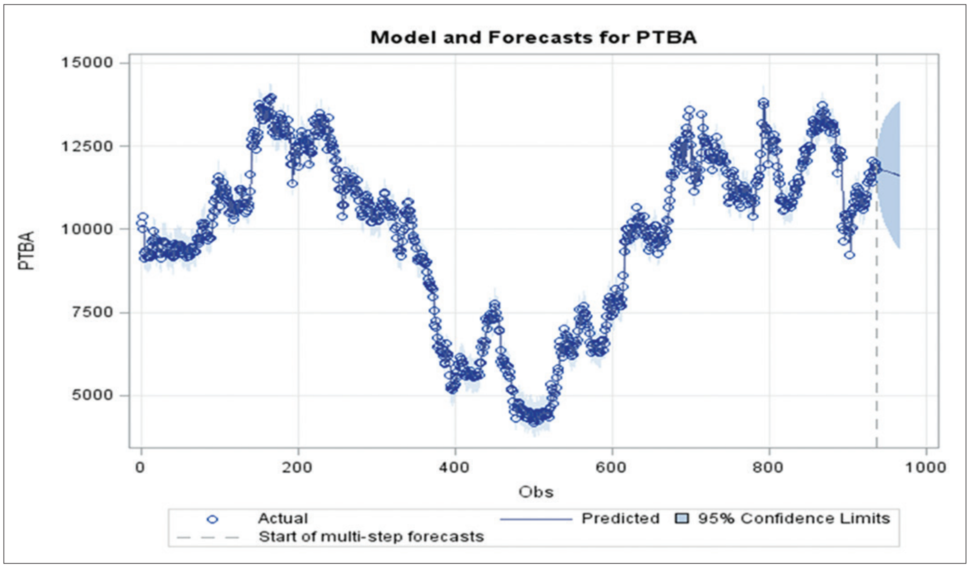
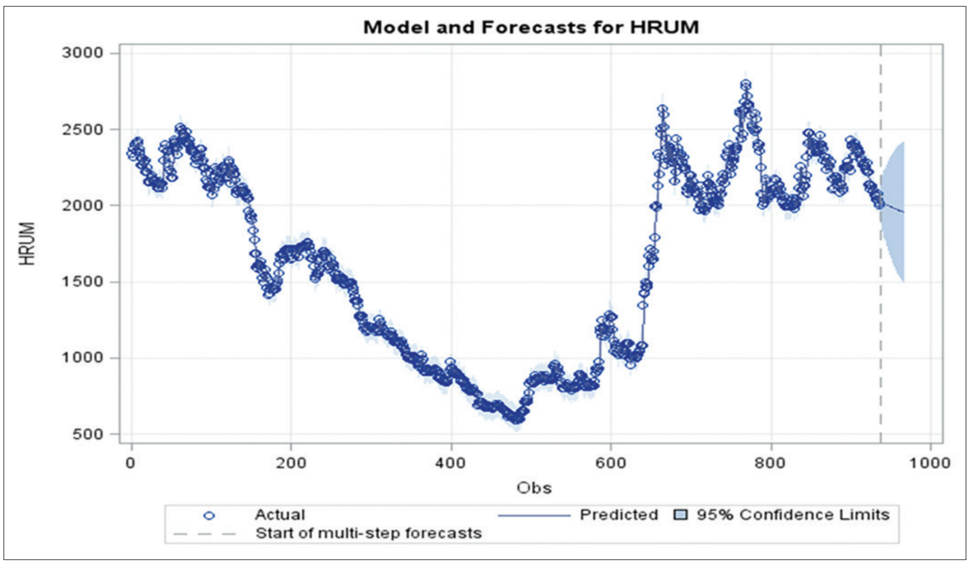


Figure 7: Prediction and forecasting of HRUM energy data



The impact up to the 6th month is positive and has significance as zero is not included in the confidence interval. However, from the 6th month up to 2 years, the impact is positive, but with no significance as zero is included in the confidence interval (Figure 5b).

3.2. Granger Causality

Table 6 shows that the Exchange rate does not induce Granger causality for PTBA and HRUM energy. The test is not significant with P value 0.7171 (>0.05). Yet, a null hypothesis is not rejected, and this result that is in line with those displayed in Table 4, where the XL0_1_1 and XL0_2_1 parameter, whose P values are 0.5935 and 0.6246, respectively, are not significant. In Table 6, Test 2 indicates that PTBA falls under the Granger causality for HRUM energy with P < 0.0001, whereas Test 3 shows that the HRUM energy does not obey the Granger causality rule for PTBA with P values of 0.8079.

Table 6: Granger causality Wald test

Test	Group	DF	Chi-square	P value
Test 1	Group 1 variables: Exchange_rate Group 2 variables: PTBA, HRUM	6	3.70	0.7171
Test 2	Group 1 variables: PTBA Group 2 variables: HRUM energy	3	23.79	<0.0001
Test 3	Group 1 variables: HRUM energy Group 2 variables: PTBA	3	0.97	0.8079

3.3. Forecasting

Forecasting is a process that allows the estimation of an unknown future value that is used in predicting the future values in a time series of data. In this study, the VARX (3,0) model was used to forecast the 12 values (1 year prior) of PTBA and HRUM energy data. Figure 6. A shows that the VARX (3,0) model for PTBA fits very well with real data. The circle represents real data and the line represents the model. The predicted values and the confidence interval of 95% are given. Accordingly, the forecasting data of

PTBA for the next 30 days seems to slightly increase. Figure 7 shows that the VARX (3,0) model for HRUM energy also fits very well with real data. As for the PTBA data, the circle represents the real values whereas the line represents the model. The predicted values and the confidence interval of 95% are given. It can be affirmed that the forecasting data of HRUM energy for the next 30 days also seems to increase.

4. CONCLUSION

Based on the results of the analysis of the relationship between the endogenous (PTBA and HRUM energy) and exogenous variables (Exchange rate), the VARX (3,0) model was found to be the best model for the relationship among these variables. The univariate models deduced from the VARX (3,0) model are very significant. On the basis of the IRF analysis, it was concluded that, if there is a shock in the Exchange rate, then the shock of one standard deviation in the Exchange rate causes PTBA and HRUM energy to produce a negative response up to 2 years before reaching a stable state (zero effect). Shock of one standard deviation in PTBA causes PTBA to respond positively and attain a stable condition after the 7th month. It seems that the impulse in PTBA has an effect on the volatility of HRUM energy; however, the HRUM energy is still stable and close to zero when a shock in PTBA occurs. Shock of one standard deviation in HRUM energy causes PTBA to respond negatively for about 2 weeks, but after that the impact is positive up to 2 years. The behavior of the confidence interval showed that the volatility is very high.

Therefore, it can be concluded that in this horizon, the closing price after the shock of HRUM energy fluctuates greatly. The impact of impulse in HRUM energy causes HRUM energy to respond positively and tend to zero in 2 years.

5. ACKNOWLEDGMENTS

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REFERENCES

Al-hajj, E., Al-Mulali, U., Solarin, S.A. (2017), The influence of oil price shocks on stock market returns: Fresh evidence from Malaysia. International Journal of Energy Economics and Policy, 7(5), 235-244.

Bank Indonesia, Foreign Exchange Transaction Rates. Available from: https://www.id.investing.com/currencies/usd-idr-historical-data. [Last retrieved on 2018 Feb 26].

Brockwell, P.J., Davis, R.A. (2002), Introduction to Time Series and Forecasting. 2nd ed. New York: Springer-Verlag.

Fuller, W.A. (1985), Nonstationary autoregressive time series. In: Hannan, E.J., Krishnaiah, P.R., Rao, M.M., editors. Handbook of Statistics. Vol. 5. Amsterdam: Elsevier Science Publishers. p1-23.

Gourieroux, C., Monfort, A. (1997), Time Series and Dynamic Models. United Kingdom: Cambridge University Press.

Hamilton, H. (1994), Time Series Analysis. Princeton, New Jersey: Princeton University Press.

Juselius, K. (2006), The Cointegrated VAR Model: Methodology and Applications. Oxford: Oxford University Press.

Kilian, L. (2011), Structural Vector Autoregressions. Ann Arbor: University of Michigan and CEPR.

Kirchgassner, G., Wolters, J. (2007), Introduction to Modern Time Series Analysis. Berlin: Springer-Verlag.

LQ45a, Data Historis Saham PTBA. Available from: https://www.seputarforex.com/saham/data_historis/index.php?kode=PTBA. [Last retrieved on 2018 Feb 25].

LQ45b, Data Historis Saham HRUM Energy. Available from: https://www.seputarforex.com/saham/data_historis/index.php?kode=HRUM. [Last retrieved on 2018 Feb 20].

Lutkepohl, H. (2005), New Introduction to Multiple Time Series Analysis. Berlin: Springer-Verlag.

Pena, D., Tiao, G.C., Tsay, R.S. (2001), A Course in Time Series Analysis. New York: John Wiley and Sons.

Sharma, A., Giri, S.S., Vardhan, H., Surange, S., Shetty, R., Shetty, V. (2018), Relationship between crude oil prices and stock market: Evidence from India. International Journal of Energy Economics and Policy, 8(4), 331-337.

Sims, C.A. (1980), Macroeconomics and reality. Econometrica, 48(1), 1-48.

Tsay, R.S. (2005), Analysis of Financial Time Series. New Jersey: John Wiley and Sons, Inc.

Tsay, R.S. (2014), Multivariate Time Series Analysis. New Jersey: John Wiley and Sons, Inc.

Wei, W.W.S. (1990), Time Series Analysis: Univariate and Multivariate Methods. Redwood City, California: Addison-Wesley Publishing Company.