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Asymmetric GARCH Value-at-Risk over MSCI in Financial Crisis

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ABSTRACT

This paper uses four asymmetric generalized autoregressive conditional heteroskedasticity (GARCH) models, which are GJR-GARCH, NA-GARCH, Threshold GARCH (T-GARCH), and AV-GARCH to compare their performance on value-at-risk (VaR) forecasting to the symmetric GARCH model. In addition, we adopt four different mean equations which are autoregressive moving average (ARMA[1,1]), AR(1), MA(1), and "in-mean" to find out a more appropriate GARCH method in estimating VaR of MSCI World Index in financial crisis. We pick up 900 daily information of MSCI World Index from 2006 to 2009. We find that GARCH-in-mean (GARCHM[1,1]), MA-GARCHM(1,1), AR(1)-T-GARCHM(1,1), and ARMA(1,1)-T-GARCHM(1,1) outperform other models in terms of number of violations. ARMA(1,1)-T-GARCHM(1,1) performs the best in terms of mean violation range, mean violation percentage, aggregate violation range, aggregate violation percentage, and max violation range. Other than T-GARCH models, number of violations decrease by using in-mean or MA(1) mean equation. Generally speaking, the better the performance in terms of violation, the larger the capital requirement is needed.

Keywords: Market Risk, Value-at-Risk, GARCH, MSCI, Financial Crisis
JEL Classifications: G2, G21

1. INTRODUCTION

Financial environment has changed more rapidly in recent years. Because of the financial innovation and deregulation, financial institutions can do more complicated trading and increase the frequency of trading activities than before. Accompanying with the financial crisis which began with the subprime market meltdown in summer 2007 and culminated with the bankruptcy of Lehman Brothers Holdings Inc., the importance of market risk management is getting more and more important (Mighri and Mansouri, 2013).

Value-at-Risk (VaR) is a tool for market risk management introduced in late 1980s. In 1993, Group of thirty recommended that VaR should be regarded as the standard of market risk measurement (Jorion, 2006). Since the New Basel Accord in 2004 formulated more sophisticated rules in capital adequacy regulation, VaR had been used more and more widely in the world.

Among various models, generalized autoregressive conditional heteroskedasticity (GARCH) models are widely used in estimating VaR through a large number of empirical studies and have turned into a more innovative method in this issue (Berkowitz and O'Brien, 2002; Andersen et al., 2013). Some previous studies have approved the ability of GARCH models to estimating VaR (e.g. Su et al., 2011; Rejeb et al., 2012). In this paper, we examine the effectiveness of GARCH models in estimating VaR during the financial crisis.

We pick MSCI World Index as our observation to find its VaR and the related risk control ability from different GARCH models. The MSCI World is a stock market index of 1500 "world" stocks. It is maintained by MSCI Inc., formerly Morgan Stanley Capital International, and is often used as a common benchmark for "world" or "global" stock funds. The index includes a collection of stocks of all the developed markets in the world, including securities from 23 countries. As a result, it is beyond all doubt to obtain more understanding to know the risk and volatility aspects of it.

We conduct our research with different types of GARCH models including symmetric GARCH-in-mean (GARCHM) model and four asymmetric GARCH models, which are Glosten-Jagannathan-Runkle GARCH (GJR-GARCH), nonlinear Asymmetric GARCH (NA-GARCH), Threshold GARCH (T-GARCH), and absolute value GARCH (AV-GARCH) to make thorough comparisons in GARCH VaR performance. In addition to the effect from different conditional variance equations based of different GARCH models, we introduce four forms of mean equations to see how the change of mean equation affects VaR performance. We conduct the forward test with indicators such as mean violation or aggregate violation to find out the best fitted model for MSCI World Index during the financial crisis. The empirical results show that the symmetric GARCHM(1,1) and MA(1)-GARCHM(1,1) produce the least number of violations. However, these two models also require more capital reservation than others. The tradeoff between conservativeness and more capital requirement could lead to further research.

The remainder of this paper is structured as follows: Section 2 offers the basic background of MSCI World Index along with the data profile and formalization and presents the major methodology of the VaR forecast models we use. Section 3 evaluates model performance with realized P & L. Section 4 gives out the conclusions.

2. DATA AND METHODOLOGY

2.1. Data

The MSCI World is a stock market index of 1500 “world” stocks. It is maintained by MSCI Inc., formerly Morgan Stanley Capital International, and is often used as a common benchmark for “world” or “global” stock funds.

The index includes a collection of stocks of all the developed markets in the world, as defined by MSCI. The index includes securities from 23 countries but excludes stocks from emerging economies making it less worldwide than the name suggests. A related index, the MSCI All Country World Index, incorporated both developed and emerging countries.

As of May 2010, the MSCI World Index consisted of the following 23 developed market country indices: Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hong Kong, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the United Kingdom, and the United States.

Since we are interested in the effectiveness of GARCH models in estimating VaR during the financial crisis, we choose September, 15, 2008, the date Lehman Brothers Holdings Inc. filed for Chapter 11 bankruptcy, to distinguish the in-sample groups and the out-of-sample group. The out-of-sample group consists of 300 observations after the date and the in-sample group comprises 600 observations before the date. As a result, we pick up 900 daily information of MSCI World Index from May 29, 2006 to Nov 6, 2009. The mean of returns for MSCI is -0.02%. The standard deviation is 1.44%. The out-of-sample group is used to test the model appropriateness and performance and to see which GARCH model can capture daily P&L change more accurately.

2.2. Methodology

In this paper, we want to compare the effectiveness in estimating VaR over MSCI World Index during the financial crisis between symmetric GARCH model and asymmetric GARCH models. So we choose the symmetric GARCHM models first as one of the evaluating models. In addition, we consider GJR-GARCH model, NA-GARCH model, T-GARCH model, and AV-GARCH model as the representatives of asymmetric GARCH models because between asymmetric models we also want to find out what kind of asymmetric model is the best fitting model for the MSCI World Index during the financial crisis.

In addition, no matter which GARCH model we use, we would also like to take additional outside effect such as transitory shock and long-term shock into consideration. As a result, we adopt three different forms of the mean equations including AR(1), MA(1) and autoregressive moving average (ARMA[1,1]) into those five GARCH VaR models we picked up respectively. AR(1) means autoregressive process, MA(1) contains moving-average model, and ARMA(1,1) considers both factors together. From different mean equations, we can see how it affects return estimates. In the next sub-chapters, we would make introductions about those five models plus the mixture of the three different mean equations applied in this paper.

In order to build GARCH VaR forecast models, the daily P&L data from in-sample group were used to estimate parameters. After forming the appropriate equations, the daily P&L data from out-sample group can be put into those equations to make the calculation of the returns and have the final estimation of the VaR forecasts under 95% and 99% confidence level. Through the process of comparing the forward testing, we can find out the best fitting VaR model for the MSCI World Index.

2.2.1. Symmetric GARCHM(1,1)

GARCHM model is widely applied in various empirical studies. It is a GARCH model adding a different mean equation which was suggested by Engle et al. (1987) that combined the conditional variance to the conditional mean equation. This characteristic made GARCHM widely used in financial time series. GARCHM(1,1) with different types of mean equations can be expressed as follows:

(1) GARCHM(1,1)

$$\begin{aligned} R_t &= \alpha + \beta * h_t^{\frac{1}{2}} + \varepsilon_t \\ h_t &= A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 \end{aligned} \tag{1}$$

(2) AR(1)-GARCHM(1,1)

$$\begin{aligned} R_t &= \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + \varepsilon_t \\ h_t &= A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 \end{aligned} \tag{2}$$

(3) MA(1)-GARCHM(1,1)

$$\begin{aligned} R_t &= \alpha + \beta * h_t^{\frac{1}{2}} - MA * \varepsilon_{t-1} + \varepsilon_t \\ h_t &= A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 \end{aligned} \tag{3}$$

(4) ARMA(1,1)-GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} - MA * \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 \quad (4)$$

where R_t is realized return at time t, h_t is the conditional variable at time t, and ε_t is the residual at time t which is a sequence of independent and identically distributed random variables with mean zero and variance h_t.

2.2.2. Innovation-shifted asymmetric NA-GARCHM(1,1)

Engle and Ng (1993) first suggested non-linear asymmetric GARCH model and the news impact curve was proposed as a measure of how news can be drawn into as the features of volatility estimates not only by traditional GARCH model but also the other parametric models which has the ability to capture the leverage and size effects. Hentschel (1995) observed that the shift conducts the major components of asymmetry and the asymmetry brought about by the shift is most pronounced for small shocks. For extremely large shocks, the asymmetric effect becomes a negligible part of the total response. The NA-GARCHM(1,1) with different types of mean equations can be expressed as follows:

(1) NA-GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + \varepsilon_t$$
$$h_t = A + B(1)\ h_{t-1} + C(1) * (\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}})^2 \quad (5)$$

(2) AR(1)-NA GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + \varepsilon_t$$
$$h_t = A + B(1)\ h_{t-1} + C(1) * (\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}})^2 \quad (6)$$

(3) MA(1)-NA GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} - MA * \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = A + B(1)\ h_{t-1} + C(1) * (\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}})^2 \quad (7)$$

(4) ARMA(1,1)-NA GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} - MA * \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = A + B(1)\ h_{t-1} + C(1) * (\varepsilon_{t-1} + C(2)\sqrt{h_{t-1}})^2 \quad (8)$$

where C(2) is a parameter that represents the innovation-shifted asymmetric effect. When C(2)<0, the negative innovations will bring about higher volatility than positive innovations of the same magnitude will do and vice versa.

2.2.3. Innovation-rotated asymmetric GJR-GARCHM(1,1)

Glosten et al. (1993) developed the GJR-GARCH model to examine the inter-temporal relation between risk and return and finished some modifications such as: Seasonal patterns in volatility, positive and negative unanticipated returns having different impacts on the conditional variance, and nominal interest rates to predict conditional variance. They found that positive unanticipated returns easily brought about a downward revision of the conditional volatility whereas negative unanticipated returns resulted in an upward revision of conditional volatility. The GJR-GARCHM(1,1) with different types of mean equations can be expressed as follows:

(1) GJR-GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + \varepsilon_t$$
$$h_t = A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 + C(1) * C(2) * |\varepsilon_{t-1}| * \varepsilon_{t-1} \quad (9)$$

(2) AR(1)-GJR GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + \varepsilon_t$$
$$h_t = A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 + C(1) * C(2) * |\varepsilon_{t-1}| * \varepsilon_{t-1} \quad (10)$$

(3) MA(1)-GJR GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} - MA * \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 + C(1) * C(2) * |\varepsilon_{t-1}| * \varepsilon_{t-1} \quad (11)$$

(4) ARMA(1,1)-GJR GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} - MA * \varepsilon_{t-1} + \varepsilon_t$$
$$h_t = A + B(1) * h_{t-1} + C(1) * \varepsilon_{t-1}^2 + C(1) * C(2) * |\varepsilon_{t-1}| * \varepsilon_{t-1} \quad (12)$$

where C(2) is a parameter that represents the innovation-rotated asymmetric effect. If C(2) is negative, the negative residual will lead to a greater impact on conditional variance than the positive one.

2.2.4. Innovation-rotated asymmetric T-GARCHM(1,1)

The Threshold-GARCH model introduced by Zakoian (1994) allows the conditional standard deviation to depend upon the sign of the lagged innovations. The T-GARCHM(1,1) with different types of mean equations can be expressed as follows:

(1) T-GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + \varepsilon_t$$
$$\sqrt{h_t} = A + B(1) * \sqrt{h_{t-1}} + C(1) * (|\varepsilon_{t-1}| + C(2) * \varepsilon_{t-1}) \quad (13)$$

(2) AR(1)-T GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + \varepsilon_t$$
$$\sqrt{h_t} = A + B(1) * \sqrt{h_{t-1}} + C(1) * (|\varepsilon_{t-1}| + C(2) * \varepsilon_{t-1}) \quad (14)$$

(3) MA(1)-T GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + MA * \varepsilon_{t-1} + \varepsilon_t \\ \sqrt{h_t} = A + B(1) * \sqrt{h_{t-1}} + C(1) * (|\varepsilon_{t-1}| + C(2) * \varepsilon_{t-1}) \quad (15)$$

(4) ARMA(1,1)-T GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + MA * \varepsilon_{t-1} + \varepsilon_t \\ \sqrt{h_t} = A + B(1) * \sqrt{h_{t-1}} + C(1) * (|\varepsilon_{t-1}| + C(2) * \varepsilon_{t-1}) \quad (16)$$

where C(2) is a parameter that represents the innovation-rotated asymmetric effect. If C(2) is negative, the negative residual will cause a larger impact on the conditional variance than the positive residual of equal amount.

2.2.5. Innovation-rotated-and-shifted asymmetric AV-GARCHM(1,1)

AV-GARCH model was proposed by Taylor (1986) and Schwert (1989). AV-GARCHM model considers both shift and rotation effect to news shock. The AV-GARCHM(1,1) with different types of mean equations can be expressed as follows:

(1) AV-GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + \varepsilon_t \\ h_t = A + B(1) * h_{t-1} + C(1) * \left[(|\varepsilon_{t-1} + \rho|) + C(2) * (\varepsilon_{t-1} + \rho) \right] \quad (17)$$

(2) AR(1)-AV GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + \varepsilon_t \\ h_t = A + B(1) * h_{t-1} + C(1) * \left[(|\varepsilon_{t-1} + \rho|) + C(2) * (\varepsilon_{t-1} + \rho) \right] \quad (18)$$

(3) MA(1)-AV GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + MA * \varepsilon_{t-1} + \varepsilon_t \\ h_t = A + B(1) * h_{t-1} + C(1) * \left[(|\varepsilon_{t-1} + \rho|) + C(2) * (\varepsilon_{t-1} + \rho) \right] \quad (19)$$

(4) ARMA(1,1)-AV GARCHM(1,1)

$$R_t = \alpha + \beta * h_t^{\frac{1}{2}} + AR * R_{t-1} + MA * \varepsilon_{t-1} + \varepsilon_t \\ h_t = A + B(1) * h_{t-1} + C(1) * \left[(|\varepsilon_{t-1} + \rho|) + C(2) * (\varepsilon_{t-1} + \rho) \right] \quad (20)$$

where C(2) is a parameter that represents the innovation-rotated asymmetric effect while ρ represents the innovation-shifted asymmetric effect.

3. EMPIRICAL RESULTS

3.1. Model Robustness and Parameter Estimates

Before we make more detailed observations to the equations formed and the VaR forecasts, we have to know if our data fit the GARCH model or not. In the paper, we conduct likelihood-ratio test to make the robustness check through comparing the maximum likelihood value before and after adopting the GARCH model (L1 and L2). If the LR (L2-L1) is greater than the critical value of Chi-square, then we could make the conclusion that the model is suitable for the data. The LRs are much larger than the critical value of Chi-square under both 95% and 99% confidence level, verifying all GARCH models applies to the MSCI World Index during the financial crisis (The results are available upon request).

Parameter estimates of various GARCH models are shown from Table 1. The coefficient α represents the mean return. The sign of α varies with different GARCH models and mean equations. However, the coefficient α are all insignificant. The insignificance of α reveals that the mean return can be fully explained by other parameters in the mean equation and proves that the model is well fitting the portfolio to estimate a proper value of VaR.

The coefficient MA is a moving average factor representing the short-term serial correlation effect. It can be regarded as the effect of short-term economic events or short-term economic events. MA(1) controls a sudden shock that will die out after 1 period. The values of MA are all positive, representing bad news of the last period will leads to downward revise of return of this period and vice versa. The values of MA are all significant under 95% confidence level except for MA(1)-NA-GARCHM(1,1) and ARMA(1,1)-T-GARCHM(1,1).

The coefficient AR is the coefficient of autoregressive process and AR(1) mean equation puts it into the model which directs the effect on return by the last period return. It is a long memory effect. If the coefficient is significant, it means the market is not efficient. The values of AR are negative and significant under 99% confidence level in GJR-GARCH models and especially symmetric GARCHM models, which means the negative return of MSCI World Index today could lead to positive return tomorrow and vice versa. The values of AR are not significant under 95% confidence level in NA-GARCH model, T-GARCH model, and AV-GARCH model.

Parameter β represents the risk premium and are all insignificant in all of our GARCH models, showing that the risk premium does not have strong influence on returns of MSCI World Index. Interestingly, β are negative under some models, indicating that higher volatility in period t will results may result in lower rate of return in period t.

In conditional variance equation, the value of A means the average volatility level. Therefore, the value of A must be positive because volatility must be a positive value. The values of A are all positive and significant under all of our models, indicating that the volatility cannot be fully explained by other factors in conditional variance equation.

The values of B(1) as the meaning of coefficient of the conditional variance in the last period all keep a large number ranging from 0.8663 to 0.9538 and are all significant under 95% or 99% confidence level, thus we find that the conditional variance is highly affected by the previous one and is able to adjust itself

Table 1: Parameter estimates of various GARCH models

Panel A estimates in GARCHM models				
	GARCHM (1,1)	**AR (1)-GARCHM (1,1)**	**MA (1)-GARCHM (1,1)**	**ARMA (1,1)-GARCHM (1,1)**
α	6.63E-04	2.19E-05	3.72E-04	–5.92E-04
A	2.28E-06**	2.30E-06**	2.19E-06**	2.26E-06**
B (1)	0.8662**	0.949292**	8.71E-01**	0.94819**
MA	0	0	0.119729**	1.16E-01*
C (1)	1.07E-01**	2.03E-02	1.03E-01**	2.16E-02
C (2)	0	0	0	0
AR	0	–0.0012918**	0	–1.26E-03**
β	–0.0377	–0.00682369	–1.46E-03	6.96E-02
Panel B estimates in NA-GARCH models				
	NA-GARCHM (1,1)	**AR (1)-NA-GARCHM (1,1)**	**MA (1)-NA-GARCHM (1,1)**	**ARMA (1,1)-NA-GARCHM (1,1)**
α	–3.98E-04	2.34E-05	–1.49E-03	–5.93E-04
A	1.58E-06**	2.31E-06**	1.97E-06**	2.26E-06**
B (1)	0.894497**	0.949227**	0.884955**	0.948138**
MA	0	0	7.18E-02	0.115818*
C (1)	3.97E-02**	2.03E-02	4.25E-02**	2.16E-02
C (2)	–0.999944*	–1.72E-02	–0.999929*	2.44E-03
AR	0	–1.29E-03	0	–1.26E-03
β	3.37E-02	–7.13E-03	0.136195	6.97E-02
Panel C estimates in GJR-GARCH models				
	GJR-GARCHM (1,1)	**AR (1)-GJR-GARCHM (1,1)**	**MA (1)-GJR-GARCHM (1,1)**	**ARMA (1,1)-GJR-GARCHM (1,1)**
α	–9.18E-04	8.58E-05	–4.22E-05	–4.06E-04
A	2.21E-06*	2.07E-06**	1.49E-06**	2.03E-06**
B (1)	0.940206**	0.953849**	0.937584**	0.952978**
MA	0	0	0.128666**	0.110703*
C (1)	2.86E-02	1.80E-02	3.22E-02*	1.92E-02
C (2)	–0.99998	–0.99987	–0.9997	–0.99995
AR	0	–1.02E-03**	0	–1.00E-03**
β	0.076179	–2.04E-02	–1.10E-02	4.51E-02
Panel D estimates in T-GARCH models				
	T-GARCHM (1,1)	**AR (1)-T-GARCHM (1,1)**	**MA (1)-T-GARCHM (1,1)**	**ARMA (1,1)-T-GARCHM (1,1)**
α	9.67E-04	–1.12E-03	4.30E-04	–1.04E-03
A	3.32E-04**	3.94E-04**	3.24E-04**	3.71E-04**
B (1)	0.931886**	0.912387**	0.932231**	0.916121**
MA	0	0	0.118267**	0.056494
C (1)	0.036864*	0.069212**	0.036368*	0.068843**
C (2)	–0.99983	–0.99836	–0.99996	–0.99881
AR	0	0.144008	0	1.29E-01
β	–0.17905	0.10073	–1.26E-01	8.27E-02
Panel E estimates in AV-GARCH models				
	AV-GARCHM (1,1)	**AR (1)-AV-GARCHM (1,1)**	**MA (1)-AV-GARCHM (1,1)**	**ARMA (1,1)-AV-GARCHM (1,1)**
α	1.02E-03	9.32E-04	6.01E-04	2.89E-04
A	3.11E-04**	3.09E-04**	3.10E-04**	3.13E-04**
B (1)	0.919409**	0.935353**	0.919955**	0.941428**
MA			0.0922848*	0.121627**
C (1)	0.0645877**	0.0445738*	0.0625128**	0.0366578*
C (2)	–0.999867**	–0.99975	–0.999986**	–0.999955
Rho	0.00114162	1.20E-03	8.91E-04	2.19E-03
AR	–0.125236	–0.0385575		–5.02E-02
β	1.02E-03	–0.122612	–7.35E-02	–3.66E-02

GARCH: Generalized autoregressive conditional heteroskedasticity, GARCHM: GARCH-in-mean, AV-GARCH: Absolute value GARCH, ARMA: Autoregressive moving average, NA-GARCH: Nonlinear asymmetric GARCH, GJR-GARCH: Glosten-Jagannathan-Runkle GARCH, T-GARCH: Threshold GARCH, *1%, **5%

from time to time. It verifies the volatility clustering phenomenon noticed by Mandelbrot (1963) in asset price.

Parameter C(1) represents how random shock in last period affects the volatility in this period. The values of C(1) are all positive, which means if the realized return in the last period was less than the expected return, it will cause the lower volatility in this period and vice versa. Generally speaking, C(1) are more significant under the mean equations of in-mean and MA(1), and is insignificant under 95% confidence level in AR(1)-NA-GARCHM(1,1), ARMA(1,1)-NA-GARCHM(1,1), AR(1)-GJR-GARCHM(1,1), ARMA(1,1)-GJR-GARCHM(1,1), AR(1)-GARCHM(1,1), and ARMA(1,1)-GARCHM(1,1).

The values of C(2) in four asymmetric models represent the asymmetric leverage effect we assumed. Except for AR(1)-NA-

GARCHM(1,1) and ARMA(1)-NA-GARCHM(1,1), the values of C(2) are all negative and very close to –1. However, The values of C(2) are only significant under the 95% confidence level in NA-GARCHM(1,1) and MA(1)-NA-GARCHM(1,1). A negative value of C(2) in NA-GARCH model brings a rightward shift of the news impact curve.

The values of ρ in AV-GARCH models represent the innovation-shifted asymmetric effect and the values of ρ are all positive but insignificant under 95% confidence level in AV-GARCH models.

Although some of the parameters are insignificant, we still use those estimates to do the out-of-sample test because the key parameter B(1) is significant and close to 1 and B(1) plus C(1) is smaller than one indicating the stability of GARCH. We will show our forward test results in the next sections.

3.2. Out-of-Sample Test under Different GARCH Models

In Tables 2-5, we present the summary of the forward testing related to the VaR forecasts, and the comparison of the ability of risk management of five GARCH models and the more detailed statistics in out-sample testing can be observed clearly through these tables as well. In this paper, we compare various indicators including violation numbers, violation rates, mean VaR, aggregate violation, maximum violation, and mean violation to examine the ability of risk management of five GARCH models.

Because of some outliner of MSCI World Index after the bankruptcy of Lehman Brothers Holdings Inc., the GARCH VaR cannot fully capture the market risk. In Table 2, under 95% confidence level, the violation rate ranges from 9% to 13.33%, while the violation rates fall into the ranges from 4.33% to 7.33% under 99% confidence level. The number of violations all models exceed the maximum number allowed in Basel Accord. It demonstrates the unpredictability of market risk during the financial tsunami, especially after the bankruptcy of Lehman Brothers Holdings Inc.

The symmetric GARCHM(1,1) model and MA(1)-GARCHM(1,1) model has better performance than asymmetric models since both model has 27 violations under 95% confidence level and 12 violations under 99% confidence level, which is inconsistent with the previous findings. Other than T-GARCH models, number of violations decreased by using in-mean or MA(1) mean equation under other GARCH models. In fact, in terms of mean equations of AR(1) and AR(1,1), the asymmetric AR(1)-T-GARCHM(1,1) model and ARMA(1,1)-T-GARCHM(1,1) model has better performance than other models. Both models have 27 violations under 95% confidence level and 14 violations under 99% confidence level.

Although GJR-GARCH models are not the best-fitting model, its performances are more consistent in all different mean equations. Indeed, the number of violations is less in GJR-GARCH model than AV-GARCH model under four different mean equations.

We also take a look at other crucial indicators such as mean violation, max violation, and aggregate violation in terms of range and percentage to specify the effectiveness of model and the efficiency of capital charge between these VaR models. In the beginning, mean violation range characterizes the average amount of additional capital charge the calculated under a certain model, and mean violation percentage can express the degree of the additional

Table 2: Mean VaR in GARCH models

$$\text{Mean VaR}=\frac{1}{n}\sum_{i=1}^{n}\text{VaR}_i$$

	Under 95% confidence level			Under 99% confidence level		
	Violation		Mean	Violation		Mean
	No.	Rate (%)	VaR (%)	No.	Rate (%)	VaR (%)
GARCHM (1,1)	27	9.00	–2.94	13	4.33	–4.16
AR (1)-GARCHM (1,1)	35	11.67	–2.40	22	7.33	–3.39
MA (1)-GARCHM (1,1)	27	9.00	–2.95	12	4.00	–4.20
ARMA (1,1)-GARCHM (1,1)	35	11.67	–2.39	22	7.33	–3.41
NA-GARCHM (1,1)	30	10.00	–2.58	16	5.33	–3.67
AR (1)-NAGARCHM (1,1)	31	10.33	–2.45	20	6.67	–3.47
MA (1)-NAGARCHM (1,1)	31	10.33	–2.63	15	5.00	–3.72
ARMA (1,1)-NAGARCHM (1,1)	33	11.00	–2.51	20	6.67	–3.58
GJR-GARCHM (1,1)	29	9.67	–2.65	15	5.00	–3.76
AR (1)-GJRGARCHM (1,1)	35	11.67	–2.40	22	7.33	–3.39
MA (1)-GJRGARCHM (1,1)	30	10.00	–2.67	16	5.33	–3.81
ARMA (1,1)-GJRGARCHM (1,1)	35	11.67	–2.39	22	7.33	–3.41
T-GARCHM (1,1)	37	12.33	–2.24	21	7.00	–3.12
AR (1)-T-GARCHM (1,1)	27	9.00	–2.75	14	4.67	–3.85
MA (1)- T-GARCHM (1,1)	40	13.33	–2.16	21	7.00	–3.08
ARMA (1,1)- T-GARCHM (1,1)	27	9.00	–2.82	14	4.67	–3.95
AV-GARCHM (1,1)	32	10.67	–2.58	16	5.33	–3.62
AR (1)- AV-GARCHM (1,1)	33	11.00	–2.48	18	6.00	–3.48
MA (1)- AV-GARCHM (1,1)	33	11.00	–2.54	16	5.33	–3.58
ARMA (1,1)- AV-GARCHM (1,1)	36	12.00	–2.31	19	6.33	–3.26

VaR: Value-at-risk, GARCH: Generalized autoregressive conditional heteroskedasticity, GARCHM: GARCH-in-mean, ARMA: Autoregressive moving average, NA-GARCHM: Nonlinear asymmetric GARCHM, GJR-GARCHM: Glosten-Jagannathan-Runkle GARCHM, T-GARCHM: Threshold GARCHM, AV-GARCHM: Absolute value GARCHM or minor capital charge which has been prepared under a certain model. From Tables 3-5, we find that the mean violation range and mean violation percentage and aggregate violation percentage are the smallest in ARMA(1,1)- T-GARCHM(1,1) under 95%

Table 3: Mean violation in GARCH models

$$\text{Mean Violation Range}=\frac{1}{n}\sum_{i=1}^{n}(R_i\text{-}VaR_i),\ \text{if violation occurs}$$

$$\text{Mean Violation Percentage}=\frac{1}{n}\sum_{i=1}^{n}\left(\frac{R_i\text{-}VaR_i}{VaR_i}\right),\ \text{if violation occurs}$$

	95% confidence level		99% confidence level	
	Range	Percentage	Range	Percentage
GARCHM (1,1)	–0.01083	–17.92	–0.00762	–43.47
AR (1)-GARCHM (1,1)	–0.01444	–27.08	–0.00958	–56.92
MA (1)-GARCHM (1,1)	–0.01133	–22.78	–0.00739	–45.83
ARMA (1,1)-GARCHM (1,1)	–0.01549	–28.52	–0.01000	–62.57
NA-GARCHM (1,1)	–0.01322	–26.12	–0.00853	–52.29
AR (1)-NAGARCHM (1,1)	–0.01457	–26.08	–0.00890	–57.77
MA (1)-NAGARCHM (1,1)	–0.01249	–29.37	–0.00852	–52.36
ARMA (1,1)-NAGARCHM (1,1)	–0.01369	–23.48	–0.00793	–53.94
GJR-GARCHM (1,1)	–0.01222	–26.10	–0.00784	–51.47
AR (1)-GJRGARCHM (1,1)	–0.01440	–26.92	–0.00953	–56.69
MA (1)-GJRGARCHM (1,1)	–0.01189	–23.76	–0.00706	–49.83
ARMA (1,1)-GJRGARCHM (1,1)	–0.01550	–28.54	–0.01001	–62.60
T-GARCHM (1,1)	–0.01340	–34.99	–0.01104	–57.86
AR (1)-T-GARCHM (1,1)	–0.01107	–17.63	–0.00615	–42.61
MA (1)- T-GARCHM (1,1)	–0.01262	–35.47	–0.01112	–55.21
ARMA (1,1)- T-GARCHM (1,1)	–0.01039	–15.23	–0.00543	–39.30
AV-GARCHM (1,1)	–0.01164	–30.69	–0.00904	–51.55
AR (1)- AV-GARCHM (1,1)	–0.01245	–27.93	–0.00871	–52.09
MA (1)- AV-GARCHM (1,1)	–0.01128	–29.46	–0.00893	–48.94
ARMA (1,1)- AV-GARCHM (1,1)	–0.01302	–32.54	–0.01042	–55.30

GARCH: Generalized autoregressive conditional heteroskedasticity, GARCHM: GARCH-in-mean, AV-GARCH: Absolute value GARCH, ARMA: Autoregressive moving average, NA-GARCH: Nonlinear asymmetric GARCH, GJR-GARCH: Glosten-Jagannathan-Runkle GARCH, T-GARCH: Threshold GARCH

Table 4: Aggregate violation in GARCH models

$$\text{Aggregate Violation Range}=\sum_{i=1}^{n}(R_i\text{-}VaR_i),\ \text{if violation occurs}$$

$$\text{Aggregate Violation Percentage}=\sum_{i=1}^{n}\left(\frac{R_i\text{-}VaR_i}{VaR_i}\right),\ \text{if violation occurs}$$

	95% confidence level		99% confidence level	
	Range	Percentage	Range	Percentage
GARCHM (1,1)	–0.29229	–215.05	–0.09141	–1173.69
AR (1)-GARCHM (1,1)	–0.50554	–595.86	–0.21081	–1992.06
MA (1)-GARCHM (1,1)	–0.30597	–273.32	–0.08868	–1237.33
ARMA (1,1)-GARCHM (1,1)	–0.54226	–627.36	–0.22004	–2189.78
NA-GARCHM (1,1)	–0.39656	–417.94	–0.13641	–1568.81
AR (1)-NAGARCHM (1,1)	–0.45181	–521.51	–0.17793	–1790.95
MA (1)-NAGARCHM (1,1)	–0.38704	–440.58	–0.12785	–1623.21
ARMA (1,1)-NAGARCHM (1,1)	–0.45168	–469.50	–0.15864	–1779.86
GJR-GARCHM (1,1)	–0.35426	–391.46	–0.11753	–1492.72
AR (1)-GJRGARCHM (1,1)	–0.50384	–592.25	–0.20957	–1984.21
MA (1)-GJRGARCHM (1,1)	–0.35663	–380.13	–0.11302	–1494.89
ARMA (1,1)-GJRGARCHM (1,1)	–0.54248	–627.95	–0.22021	–2191.06
T-GARCHM (1,1)	–0.49588	–734.72	–0.23191	–2141.00
AR (1)-T-GARCHM (1,1)	–0.29885	–246.76	–0.08608	–1150.35
MA (1)- T-GARCHM (1,1)	–0.50470	–744.89	–0.23356	–2208.60
ARMA (1,1)- T–GARCHM (1,1)	–0.28064	–213.20	–0.07596	–1061.13
AV-GARCHM (1,1)	–0.37251	–491.05	–0.14456	–1649.75
AR (1)- AV-GARCHM (1,1)	–0.41090	–502.73	–0.15677	–1718.87
MA (1)- AV-GARCHM (1,1)	–0.37212	–471.38	–0.14293	–1615.11
ARMA (1,1)- AV-GARCHM (1,1)	–0.46873	–618.27	–0.19792	–1990.83

GARCH: Generalized autoregressive conditional heteroskedasticity, GARCHM: GARCH-in-mean, AV-GARCH: Absolute value GARCH, ARMA: Autoregressive moving average, NA-GARCH: Nonlinear asymmetric GARCH, GJR-GARCH: Glosten-Jagannathan-Runkle GARCH, T-GARCH: Threshold GARCH

Table 5: Max violation in GARCH models

Max Violation Range=Min(R_i-VaR_i), for i=1 to n

Max Violation Percentage=Max$\left(\frac{R_i - VaR_i}{VaR_i}\right)$, for i=1 to n

	95% confidence level		99% confidence level	
	Range	Percentage	Range	Percentage
GARCHM (1,1)	–0.04030	37.70	–0.02714	127.23
AR (1)-GARCHM (1,1)	–0.04998	131.16	–0.04084	227.17
MA (1)-GARCHM (1,1)	–0.04167	65.68	–0.02854	137.49
ARMA (1,1)-GARCHM (1,1)	–0.05102	138.36	–0.04178	243.43
NA-GARCHM (1,1)	–0.04935	124.15	–0.03987	218.05
AR (1)-NAGARCHM (1,1)	–0.05109	143.87	–0.04246	244.63
MA (1)-NAGARCHM (1,1)	–0.04985	127.64	–0.04036	225.30
ARMA (1,1)-NAGARCHM (1,1)	–0.05161	145.88	–0.04271	253.33
GJR-GARCHM (1,1)	–0.04533	90.37	–0.03417	170.09
AR (1)-GJRGARCHM (1,1)	–0.04994	130.81	–0.04079	226.66
MA (1)-GJRGARCHM (1,1)	–0.04614	93.67	–0.03481	178.60
ARMA (1,1)-GJRGARCHM (1,1)	–0.05102	138.36	–0.04178	243.44
T-GARCHM (1,1)	–0.04935	128.55	–0.04048	218.04
AR (1)-T-GARCHM (1,1)	–0.04097	64.62	–0.02826	132.10
MA (1)- T-GARCHM (1,1)	–0.04837	121.61	–0.03950	204.92
ARMA (1,1)- T-GARCHM (1,1)	–0.03989	60.21	–0.02705	124.29
AV-GARCHM (1,1)	–0.04573	95.59	–0.03518	174.20
AR (1)- AV-GARCHM (1,1)	–0.04787	112.08	–0.03804	198.54
MA (1)- AV-GARCHM (1,1)	–0.04465	88.83	–0.03386	163.40
ARMA (1,1)- AV-GARCHM (1,1)	–0.04882	122.03	–0.03956	210.82

GARCH: Generalized autoregressive conditional heteroskedasticity, GARCHM: GARCH-in-mean, AV-GARCH: Absolute value GARCH, ARMA: Autoregressive moving average, NA-GARCH: Nonlinear asymmetric GARCH, GJR-GARCH: Glosten-Jagannathan-Runkle GARCH, T-GARCH: Threshold GARCH

and 99% confidence level. In terms of maximum violation range, we can see from Table 5 that ARMA(1,1)- T-GARCHM(1,1) is the smallest under 95% and 99% confidence level. In terms of maximum violation percentage, ARMA(1,1)- T-GARCHM(1,1) is the smallest in under 95% confidence level and GARCHM(1,1) in mean is the smallest in under 99% confidence level.

To sum up, GARCHM(1,1) in mean, MA-GARCHM(1,1), AR(1)- T-GARCHM(1,1), and ARMA(1,1)- T-GARCHM(1,1) fit better than other models for estimating VaR of MSCI World Index during the financial crisis. However, if we change the mean equations of the aforementioned models, the performance of these models will worsen greatly. Another noticeable fact is that other than T-GARCH models, number of violations decreased by using in-mean or MA(1) mean equation. Also, the financial market became so mercurial and unpredictable that all GARCH models do not fall into the safe range in terms of the regulation by Basel Accord. Overall, the empirical findings are not consistent with the previous literature.

3.3. The Tradeoff between Conservativeness and Less Capital Reserve

The VaR prediction produced by the models which have better performance in terms of violation is most of the times larger than others done. This may lead to an inference that the better the performance in terms of violation, the larger the capital requirement is needed.

We tried to capture this phenomenon by computing average absolute difference between the real return and the VaR produced by each models. We define the average absolute difference as follows:

$$\text{Average Absolute Difference} = \frac{|\text{VaR - Rt}|}{\text{Number of Observations}} \quad (21)$$

The assumption is to detect capital efficiency of smaller average absolute difference. The detailed results are included in Table 6. Generally speaking, there is a tradeoff between capital efficiency and conservativeness. However, this average absolute difference could not provide a strong evidence to draw the conclusion of real efficiency of capital reservation of each model.

4. CONCLUSION

In this paper, we have conducted GARCHM, NA-GARCH, GJR-GARCH, T-GARCH, and AV-GARCH models with four different types of mean equations to find out the most appropriate model fitted for VaR estimation over MSCI World Index. Besides, we make comparison with VaR and other related indicators under each model applying to each other. And in our empirical studies and pursuant analyses in comparison between the results from those GARCH models, we have the following major findings: First, GARCHM(1,1) in mean, MA-GARCHM(1,1), AR(1)- T-GARCHM(1,1), and ARMA(1,1)- T-GARCHM(1,1) outperform other models in terms of number of violations. Second, ARMA(1,1)- T-GARCHM(1,1) performs the best in terms of mean violation range, mean violation percentage, aggregate violation range, aggregate violation percentage, and max violation range. Third, other than T-GARCH models, number of violations decreased by using in-mean or MA(1) mean equation.

Table 6: Average absolute difference of VaR and Rt in GARCH models

$$\text{Average Absolute Difference} = \frac{|\text{VaR} - R_t|}{\text{Number of observations}}$$

Average absolute difference between VaR and Rt	Under 95% confidence level (%)	Under 99% confidence level (%)
GARCHM (1,1)	3.10	4.19
AR (1)-GARCHM (1,1)	2.69	3.49
MA (1)-GARCHM (1,1)	3.11	4.21
ARMA (1,1)-GARCHM (1,1)	2.71	3.52
GJR-GARCHM (1,1)	2.80	3.72
AR (1)- GJR-GARCHM (1,1)	2.71	3.55
MA (1)- GJR-GARCHM (1,1)	2.85	3.77
ARMA (1,1)- GJR-GARCHM (1,1)	2.77	3.64
NA-GARCHM (1,1)	2.85	3.80
AR (1)- NA-GARCHM (1,1)	2.69	3.49
MA (1)- NA-GARCHM (1,1)	2.86	3.85
ARMA (1,1)- NA-GARCHM (1,1)	2.71	3.52
T-GARCHM (1,1)	2.53	3.23
AR (1)-T-GARCHM (1,1)	2.94	3.99
MA (1)- T-GARCHM (1,1)	2.49	3.17
ARMA (1,1)- T-GARCHM (1,1)	2.99	4.06
AV-GARCHM (1,1)	2.79	3.67
AR (1)- AV-GARCHM (1,1)	2.71	3.54
MA (1)- AV-GARCHM (1,1)	2.75	3.63
ARMA (1,1)- AV-GARCHM (1,1)	2.58	3.35

GARCH: Generalized autoregressive conditional heteroskedasticity,
GARCHM: GARCH-in-mean, AV-GARCH: Absolute value GARCH,
ARMA: Autoregressive moving average, NA-GARCH: Nonlinear asymmetric GARCH,
GJR-GARCH: Glosten-Jagannathan-Runkle GARCH, T-GARCH: Threshold GARCH

Fourth, generally speaking, the better the performance in terms of violation, the larger the capital requirement is needed. Lastly, all models fail to stay in the safe range in terms of the regulation by Basel Accord.

The sign of the AR coefficients may be the reason why number of violations increased by using AR(1) or ARMA(1.1) mean equations. Other than T-GARCH models, the AR coefficients are all negative, which means the negative return of MSCI World Index today could lead to positive return tomorrow and vice versa. However, after the bankruptcy of Lehman Brothers Holdings Inc., the negative return of MSCI World Index today tends to continue tomorrow because the market needs more time to digest the bad news. Therefore, when we use the negative AR coefficients in out-of sample tests, VaR tend to be underestimated, resulting in more violations in these model. In fact, the AR coefficients are positive in AR(1)-T-GARCHM(1,1) and ARMA(1,1)- T-GARCHM(1,1) models, that is why these two models outperform T-GARCHM and ARMA(1,1)-T-GARCHM models.

Another implication of this finding is that the financial environment may have changed drastically after Lehman Brothers Holdings Inc. filed for bankruptcy on September 15, 2008. Therefore, the return patterns of MSCI World Index are different before and after September 15, 2008. The estimated parameters before the date may be improper for the out-of-sample group.

Another possible reason for explaining the inconsistency of empirical findings between this paper and the papers before the financial crisis is that the MSCI World Index consists of 23 developed market country indices, and the transaction time for each country varies. This may distort the real return of MSCI World Index. Although the empirical results are a little surprising, but overall, the tradeoff between conservativeness and more capital requirement could lead to further research.

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