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Revisit Closed-End Fund Puzzles via Dynamic Capital Mobility

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ABSTRACT

A theoretic model based on the concepts of constrained arbitrage and capital mobility is proposed to interpret closed-end fund puzzles. Although a discount for a closed-end fund's price relative to its net asset value is more prevalent, our model never excludes the possibility of a premium, which depends on the relative magnitude of the key parameters for the closed-end fund and its component stocks. Since closed-end funds tend to be more owned by individual investors who are less likely to be active traders due to investor inertia, and investor enthusiasm is usually higher for stocks than for closed-end funds, the aggregated price of component stocks will be more likely higher than the price of the closed-end fund, thus leading to the discount. Our model further shows that a closed-end fund's discount is negatively related to its expected dividends and the interest rate. The above results are reproduced by simulation.

Keywords: Closed-End Fund Puzzles, Constrained Arbitrage, Capital Mobility, Investor Inertia, Investor Sentiment
JEL Classifications: G12, G14, G23, G40

1. INTRODUCTION

Our financial market is not such a perfect market as the efficient market hypothesis (EMH) assumes. We do observe that two types of assets with rather similar cashflows are priced differently, which is exactly against the prediction of our traditional valuation theory. For instance, in the U.S. fixed-income markets, off-the-run (previously issued) Treasury bonds are usually traded at a lower price than on-the-run (newly issued) comparable bonds with a similar maturity. Vayanos and Weill (2008) establish a search-based model to investigate this on-the-run phenomenon in the over-the-counter (OTC) fixed income markets.

Closed-end fund puzzles provide another vivid example in which the market price of a closed-end fund is rarely equal to its net asset value (NAV). If a closed-end fund's price is less than its NAV, the closed-end fund is traded at a discount. Meanwhile, if a closed-end fund's price is greater than its NAV, it is traded at a premium. Based on the data from CEF Connect, 77% of closed-end funds were traded at a discount in 2018[1]. Furthermore, according to the analysis by Invesco Ltd., the historical average discount for closed-end funds is -4.18% although the most recent discount value is reduced to −2.86% in September 2019[2]. Patro et al. (2017) design an investment strategy to exploit closed-end fund discounts and indicate that closed-end fund markets are much more inefficient than people thought before. Why are closed-end funds more frequently traded at a discount than at a premium? How can the power of arbitrage not eliminate the discrepancy between a closed-end fund's price and its NAV?

Lee et al., (1990) systemically address closed-end fund puzzles and provide a comprehensive review of the possible explanations of the phenomena. A closed-end fund's price is determined by supply

1 https://www.forbes.com/sites/simonmoore/2018/04/05/do-closed-end-fund-discounts-signal-opportunity-to-investors/#68a121156cd5
2 https://www.invesco.com/pdf/U-DSCNT-FLY-1-E.pdf

and demand and thus could diverge from its NAV. Specifically, standard theories including agency costs, restricted stocks, and tax liabilities have been proposed to explain the existence of a discount for closed-end funds. According to agency costs theory, the discount might either reflect a closed-end fund manager's persistent underperformance with respect to management fees or reflect the ineffective monitoring by the board of directors of closed-end funds (Boudreaux, 1973; Guercio et al., 2003). Restricted stock theory, however, considers a closed-end fund's NAV, not as its true value but overvalued if the fund holds a large amount of illiquid stock which cannot be traded freely in the open market (Malkiel, 1977). Tax liabilities-based theory attributes the discount to the capital gain tax burden borne by the existing closed-end fund shareholders even if they never make any money.

Lee et al., (1991) further suggest an alternative explanation for closed-end fund puzzles based on the concept of "noise traders" originated from De Long et al. (1990). They propose that the discount is driven by individual investor sentiment and find that both closed-end funds and small stocks are more likely held by individual investors and thus are subject to the same individual investor sentiment. However, Chen et al. (1993) find that there is a lack of co-movement between closed-end fund discounts and small firm returns and thus challenge the foundation of investor sentiment based theory.

Berk and Stanton (2007) extend the agency costs theory by combining the managerial ability with the existing labor contract in the industry, implying that a premium is short-lived and most closed-end funds are traded at a discount under the labor contract used in practice since a bad performance fund manager is usually entrenched while a good performance fund manager would threaten to quit if the pay is not increased. Cherkes et al., (2009) extend the restricted stock theory to a liquidity-based model for closed-end funds and interpret the discount as the trade-off between the liquidity benefits holding closed-end fund shares and the management fees charged by the fund managers.

Research on closed-end fund puzzles has expanded to specific closed-end funds investing in foreign assets or targeting unique investment instruments. Bonser-Neal et al. (1990) examine the relationship between international investment restrictions and closed-end country fund prices. Bodurtha et al. (1995) show that closed-end country fund premiums may be explained as the U.S. market sentiment. Nishiotis (2004) find that indirect investment barriers have a powerful effect on emerging market closed-end fund discounts/premiums. Elton et al. (2013) identify an additional benefit for investors holding closed-end bond funds in terms of leveraging their fixed-income investment at very low borrowing rates. Fletcher (2013) provides a thorough survey focusing particularly on liquidity, sentiment, and segmentation of closed-end country funds.

Even if a closed-end fund is initially mispriced, under the condition of no arbitrage assumed in traditional finance theory, intelligent arbitrageurs would make money by taking full advantage of this golden opportunity. Thus, the discrepancy between the closed-end fund's price and its NAV would gradually shrink or even disappear. For instance, for a closed-end fund traded at a discount, arbitrageurs could buy out the entire fund and liquidate it; or they could set up a long position in the fund while shorting its component stocks. However, in practice, there are multiple obstacles to prevent arbitrageurs from fully eliminating closed-end fund discounts/premiums. Bradley et al. (2010) investigate the impacts of open-ending attempts by activist arbitrageurs on closed-end fund discounts. They find that open-ending attempts can only reduce discounts to half of their original level on average. Besides transaction costs and take-over resistances by fund managers, the financial constraint of arbitrageurs can be another crucial obstacle, i.e., due to a limited amount of capital, arbitrageurs are not able to invest in all profitable arbitrage opportunities and they can only target a finite number of projects. Pontiff (1996) finds empirically that the price of a closed-end fund is more likely to deviate from the value of its underlying stocks if the arbitrage cost is higher. Gemmill and Thomas (2002) provide evidence that costly arbitrage may let asset prices deviate from fundamental values for a long time via a sample of 158 closed-end funds. Gromb and Vayanos (2018) develop a general model to investigate how financially constrained arbitrageurs exploit the price discrepancies among segmented markets. In their model, local hedgers are immobile while arbitrageurs can allocate their capital among different markets, however, subject to the financial constraint which prevents them from driving returns perfectly equal in the two markets. Duffie and Strulovici (2012) establish a search-based model to characterize the equilibrium movement of capital between partially integrated asset markets in which financial intermediaries optimally move the capital of individual investors from the "over-capitalized" market to the "under-capitalized" market.

In this paper, we revisit closed-end fund puzzles and rationalize the existence of closed-end fund discounts through a dynamic capital mobility model. Different from standard theories, our interpretation never resorts to the managerial ability and fees of closed-end funds. Our model assumes that financially constrained arbitrageurs are not able to allocate their limited capital to fully equalize the key parameters among originally partially integrated markets. Even if the markets for the closed-end fund and the underlying component stocks seem closely connected, they are still not the same market. Thus, they may attract investors with marginally different preferences if not entirely different. Weiss (1989) finds that the average institutional ownership for the sample closed-end funds is 3.50% while the corresponding number for the control group is 21.82%. Lee et al., (1991) also find that closed-end funds tend to be owned by individual investors. Moreover, Ameriks and Zeldes (2004) find that roughly 77% of households tracked over 10 years made no changes for their asset allocation and another 14% made only one change. Mitchell et al. (2006) report that based on data for 1.2 million workers in over 1500 plans, most workers are "*inattentive portfolio managers*" who don't engage in any trading for their defined contribution retirement plans. Therefore, a tiny difference of investor inertia or enthusiasm not fully erased by arbitrageurs will lead to a disparity of the key parameters in the markets for the closed-end fund and the underlying component stocks, which can ultimately produce the size of discounts currently observed in financial markets.

While the existence of a discount is more common, thus attracting much attention of researchers on closed-end funds, a premium for a closed-end fund is also possible. Based on the same data source from CEF Connect, we may infer that about 23% (≈1-77%) of closed-end funds were traded at a premium in 2018. Our model not only targets a discount but also can explain the existence of a premium. The existence of a discount or premium is ultimately determined by the empirical values of the key parameters in those markets.

Moreover, standard theories have not been able to make a difference for the roles played by various factors affecting closed-end fund puzzles. We isolate those factors whose function is to control the existence of a discount or premium from the other factors which mainly modulate the above effect. Specifically, the jumping rates between a bearish investor and a bullish investor, and the trading rates among investors belong to direction factors, controlling whether a discount or premium occurs. For instance, our model indicates that since the jumping-up rate from a bearish investor to a bullish investor for a closed-end fund is less than the corresponding rates for component stocks, ceteris, paribus, the aggregated price of component stocks will be more likely higher than the price of the closed-end fund, thus leading to a discount. Here the jumping-up rate from a bearish investor to a bullish investor to be defined in our model provides a new measure of investor sentiment. Whereas the interest rate and the expected dividends are classified as modulating factors. As an example, when expected dividends of a closed-end fund rise, its discount decreases. However, the impact of the interest rate on the discount is more complicated, which needs to be explored via simulation.

The key constituting part of our model that determines the transaction price of underlying stocks is significantly different from the traditional "frictionless" stock valuation approach centered around the dividend discount model (DDM). Our approach introduces the dynamics of type transformation of investors which can better mimic the real stock trading process. Song and Mussa (2021) apply a similar approach to investigating the trade behaviors between buyers and sellers in the commercial real estate market. Both models are originated from a general theory of asset pricing with search and bargaining proposed by Duffie et al., (2005).

The paper is organized as follows: Section 2 sets up a theoretical model for a discount or premium of a closed-end fund; Section 3 does the model calibration and simulation; Section 4 concludes the paper. Symbols and notations are summarized in Appendix A. Proofs of propositions and corollaries are provided in Appendix B.

2. THEORY

2.1. A Dynamic Model to Determine the Transaction Price of an Individual Stock

In this section, we derive a formula for the transaction price of an individual stock based on a dynamic stock trading process, which will be utilized to calculate the discount or premium of a closed-end fund later.

In the market for an individual stock, there are many investors continuously trading it. Investors are characterized by their attitude to the future price movement of the stock. An investor who is optimistic about the stock and expects that the stock's price will rise is called a bullish investor and denoted by "H" while an investor who is pessimistic about the stock and believes that the stock's price will decline is called a bearish investor and denoted by "L." Furthermore, an investor's attitude is dynamic. In other words, a bullish investor can become a bearish investor with a jumping-down rate of π_d. Meanwhile, a bearish investor can also turn into a bullish investor with a jumping-up rate of π_u.

To simplify our model, we assume that each transaction includes the exchange of only one share of the stock. Thus, an investor is also characterized by whether holding one share of the stock or not.

A bullish investor when holding one share of the stock expects to receive one unit of dividends during each period. However, a bearish investor when holding one share of the same stock expects to only obtain (1-δ) units of dividends (here $0<\delta<1$) during each period. Dividends are measured by consumption goods.

Although we assume that investors are homogenous, *ex-ante*, there are four types of them, *ex-post*, according to whether they hold one share of the stock or not, and whether they are bullish (high sentiment) or bearish (low sentiment). Thus, we define four value functions:

V_{H0}: the value of a bullish investor without one share of the stock
V_{H1}: The value of a bullish investor with one share of the stock
V_{L0}: The value of a bearish investor without one share of the stock
V_{L1}: The value of a bearish investor with one share of the stock

According to our model's structure, the only possible trading is between a bullish investor without one share of the stock (H_0) as a buyer and a bearish investor with one share of the stock (L_1) as a seller. The transaction price of one share of the stock is represented by P and the corresponding trading rates are denoted as α and β, which follow a standard Poisson process. In other words, on average, during each period a bearish investor's selling rate is α while a bullish investor's buying rate is β. Here, α (β) measures the easiness of investors to sell (buy) the stock in the market.

Figure 1 depicts the dynamic transformation of four types of investors for the stock

Lastly, we assume that time is infinite, and investors are risk-neutral with the (risk-free) discount rate r. Then, we establish four Bellman equations to solve for the transaction price of one share of the stock in market equilibrium.

$$\text{For } V_{L1}\text{: } r V_{L1}=1-\delta + \pi_u (V_{H1}-V_{L1}) + \alpha (P+ V_{Lo} - V_{L1}) \quad (1)$$

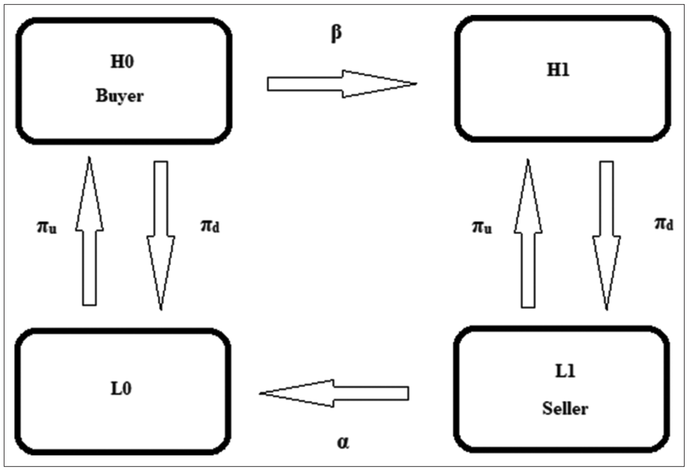
$$\text{For } V_{H0}\text{: } r V_{H0}= \pi_d (V_{L0}-V_{H0}) +\beta (V_{H1}-V_{H0}-P) \quad (2)$$

$$\text{For } V_{H1}\text{: } r V_{H1}=1 + \pi_d (V_{L1}-V_{H1}) \quad (3)$$

$$\text{For } V_{L0}\text{: } r V_{L0}= \pi_u (V_{H0}-V_{L0}) \quad (4)$$

All four equations have a similar structure: the left-hand side called the flow value is always the product of the discount rate and the corresponding value function; the right-hand side represents the value change due to the investor's attitude jump and/or the stock trading.

Figure 1: Dynamic transformation of four types of investors



For instance, in Equation (1), the left-hand side (rV_{L1}) represents the flow value for a bearish investor with one share of the stock. The right-hand side consists of three items: the first item indicates that if there is no attitude/sentiment change, the bearish investor with one share of the stock expects to receive (1-δ) units of dividends during each period; the second item means that the bearish investor with one share of the stock can become a bullish investor with a jumping-up rate of π_u; the third item shows that with a selling rate of α the bearish investor with one share of the stock can trade with a bullish investor without one share of the stock at the transaction price P.

Moreover, the transaction price of one share of the stock P is pinned down as the middle point of the surplus functions of buyers (H_0) and sellers (L_1):

$$P= [(V_{L1}-V_{L0}) +(V_{H1}- V_{H0})]/2 \tag{5}$$

Combining Equations (1-5), we solve for P as a function of (r, δ, α, β, π_d, π_u). The result is summarized in Proposition 1.

Proposition 1: For the market of an individual stock characterized by Equations (1-5), we define two surplus functions, $\Delta V_L = V_{L1}-V_{L0}$ and $\Delta V_H = V_{H1}-V_{H0}$. Thus, the transaction price of one share of the stock in market equilibrium P is:

$$P = (\Delta V_L + \Delta V_H)/2$$

$$=\frac{1}{r}-\frac{\delta}{r}\frac{(\frac{r}{2}+\pi_d+\frac{\beta}{2})}{(r+\pi_u+\pi_d+\frac{\beta}{2}+\frac{\alpha}{2})} \tag{6}$$

Intuitively, according to the standard constant dividends stock valuation model, the sellers ($_{L1}$) value one share of the stock at $\frac{1-\delta}{r}$ since they expect to receive (1-δ) units of dividends infinitely while the buyers (H_0) value the same stock at $\frac{1}{r}$ since they expect to receive one unit of dividends infinitely. However, the market equilibrium price of the stock in our model is represented by Equation (6) which falls between the above two extreme points. Here, the key difference between our stock valuation approach and the traditional dividend discount model (DDM) is that we introduce the dynamics of type transformation of investors to better mimic the real stock trading process, making it possible to study closed-end fund puzzles in next section.

2.2. The Definition of the Discount or Premium of a Closed-end Fund

We assume that a closed-end fund c only invests in two stocks, a and b, and that each share of the closed-end fund corresponds to the ownership of one share of stock a and one share of stock b. The main findings derived from this unique portfolio weight can be directly extended to any other portfolio construction.

Thus, when managerial ability and compensation of the fund manager are not considered, the net asset value (NAV) of the closed-end fund c equals $P_a + P_b$, here P_a represents the transaction price of one share of stock a and P_b represents the transaction price of one share of stock b. Meanwhile, if the transaction price of one share of the closed-end fund c is denoted by P_c, the discount (or premium) of the closed-end fund c (measured by dollars) can be expressed as P_c - ($P_a + P_b$). Applying the result from Proposition 1, we obtain:

Definition: For a closed-end fund investing in an equal share of stock a and b, the discount/premium of the closed-end fund (measured by dollars) is thus defined as:

$$P_c - NAV = P_c - (P_a + P_b)$$

$$=[\frac{2}{r}-\frac{\delta_c}{r}\frac{\left(\frac{r}{2}+\pi_{d,c}+\frac{\beta_c}{2}\right)}{\left(r+\pi_{u,c}+\pi_{d,c}+\frac{\beta_c}{2}+\frac{\alpha_c}{2}\right)}]-$$

$$\{[\frac{1}{r}-\frac{\delta_a}{r}\frac{\left(\frac{r}{2}+\pi_{d,a}+\frac{\beta_a}{2}\right)}{\left(r+\pi_{u,a}+\pi_{d,a}+\frac{\beta_a}{2}+\frac{\alpha_a}{2}\right)}]+$$

$$[\frac{1}{r}-\frac{\delta_b}{r}\frac{\left(\frac{r}{2}+\pi_{d,b}+\frac{\beta_b}{2}\right)}{\left(r+\pi_{u,b}+\pi_{d,b}+\frac{\beta_b}{2}+\frac{\alpha_b}{2}\right)}]\}$$

$$=\frac{\delta_a}{r}\frac{\left(\frac{r}{2}+\pi_{d,a}+\frac{\beta_a}{2}\right)}{\left(r+\pi_{u,a}+\pi_{d,a}+\frac{\beta_a}{2}+\frac{\alpha_a}{2}\right)}+$$

$$\frac{\delta_b}{r}\frac{\left(\frac{r}{2}+\pi_{d,b}+\frac{\beta_b}{2}\right)}{\left(r+\pi_{u,b}+\pi_{d,b}+\frac{\beta_b}{2}+\frac{\alpha_b}{2}\right)}-$$

$$\frac{\delta_c}{r}\frac{\left(\frac{r}{2}+\pi_{d,c}+\frac{\beta_c}{2}\right)}{\left(r+\pi_{u,c}+\pi_{d,c}+\frac{\beta_c}{2}+\frac{\alpha_c}{2}\right)} \tag{7}$$

Here, the subscripts of a, b, and c represent the parameters for stock a, stock b, and the closed-end fund, respectively

Since investors can freely move their capital between stock a and stock b, we consider they are traded in a fully integrated stock market. Thus, we assume that the key parameters for stock a and b are equal in market equilibrium. However, probably, investors who are trading in the market for the closed-end fund are different from those who are trading in the markets for stock a and b. Thus, we assume that the market for the closed-end fund is partially integrated with the markets for stock a and b. In other words, the parameters for stock a and b are not necessarily equal to those for the closed-end fund. Moreover, since potential cross-market arbitrageurs are typically financially constrained and cannot take advantage of all the existing opportunities in the financial market, we assume that no-arbitrage condition is not sufficient to fully flatten the difference between those parameters. Figure 2 represents the relationship between the closed-end fund c and its two components, stock a and stock b.

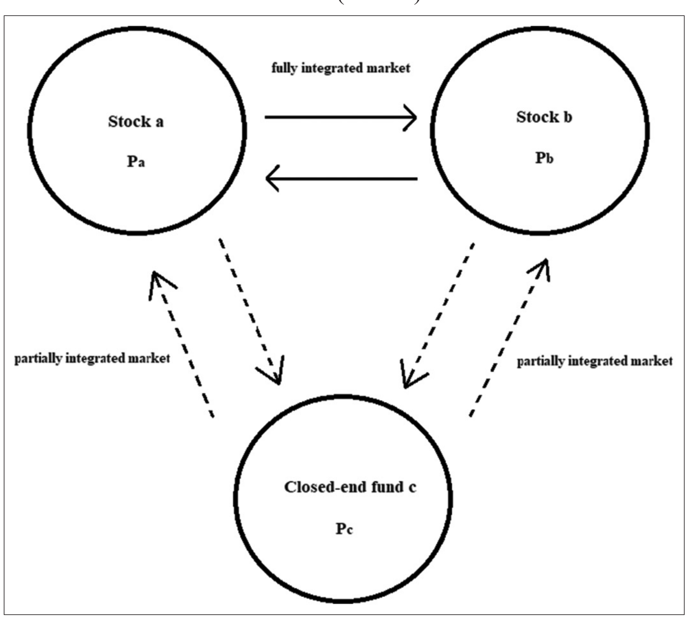
Lastly, we assume that investors' expectation on the dividends of the closed-end fund is self-consistent with its component stocks (a and b), i.e., $\delta_c = \delta_a + \delta_b$. Then Equation (7) can be simplified in Proposition 2.

Proposition 2: For a closed-end fund investing in an equal share of stock a and b, if $\pi_{d,a} = \pi_{d,b} \equiv \pi_d$, $\pi_{u,a} = \pi_{u,b} \equiv \pi_u$, and $\delta_c = X\delta_a + Y\delta_b$, the discount/premium of the closed-end fund (measured by dollars) is thus expressed as:

$$P_c - NAV = P_c - (P_a + P_b)$$

$$\frac{\delta_a + \delta_b}{r}\left[\frac{\left(\frac{r}{2} + \pi_d + \frac{\beta}{2}\right)}{\left(r + \pi_u + \pi_d + \frac{\beta}{2} + \frac{\alpha}{2}\right)} - \frac{\left(\frac{r}{2} + \pi_{d,c} + \frac{\beta_c}{2}\right)}{\left(r + \pi_{u,c} + \pi_{d,c} + \frac{\beta_c}{2} + \frac{\alpha_c}{2}\right)}\right] \quad (8)$$

Figure 2: Markets for the Closed-End Fund c and its Component Stocks (a and b)



Based on Equation (8), we can investigate the impacts of various parameters on closed-end fund puzzles. The results are summarized in Corollary 1 and 2. Here we classify those parameters into the *direction factors* (α, β, π_u, π_d) and the modulating factors (δ, r). The direction factors mainly determine whether the closed-end fund has a discount or premium while the modulating factors enhance or weaken the above effect once the discount or premium exists.

Corollary 1 (The impact of direction factors): If $\alpha > \alpha_c$, $\beta < \beta_c$, $\pi_u > \pi_{u,c}$, and $\pi_d < \pi_{d,c}$, then $P_c <$ NAV, i.e., there exists a discount for the closed-end fund.

For instance, since closed-end funds tend to be more owned by individual investors (Weiss, 1989; Lee et al., 1991) who are less likely to be active in transactions due to investor inertia (Ameriks and Zeldes, 2004; Mitchell et al. 2006), and investor sentiment or enthusiasm for stocks is usually higher than that for the closed-end fund (Lee et al., 1991; Fletcher, 2013), the key parameters in the market of the closed-end fund c and the markets of stock a and stock b may be different in two main aspects: (1) the ability to trade and (2) the willingness to trade. Thus, the selling rate for stocks which measures the easiness (ability) to sell is typically larger than that for the closed-end fund ($\alpha > \alpha_c$), and the jumping-up rate from a bearish investor to a bullish investor for stocks which reflects the willingness to trade is mostly larger than that for the closed-end fund ($\pi_u > \pi_{u,c}$). *Ceteris paribus,* the aggregated price of component stocks will be more likely higher than the price of the closed-end fund, thus leading to a discount for the closed-end fund. Following a similar logic, the other scenarios can be easily analyzed.

Corollary 2 (The impact of modulating factors): If there exists a discount for the closed-end fund, then the larger the size of , the more severe the discount of the closed-end fund. However, the impact of r is more complicated.

While the sign of Equation (8) in Proposition 2 is determined by the two items inside the brackets, and r modulate the magnitude of a discount or premium of the closed-end fund. Recalling from the previous session, $1-\delta_a$, $1-\delta_b$, $2-(\delta_a + \delta_b)$ represent the dividends expected by a bearish investor for stock a, stock b, and the closed-end fund, respectively, whereas 1, 1, and 2 represent the dividends expected by a bullish investor for stock a, stock b, and the closed-end fund. When $(\delta_a + \delta_b)$ is larger, the expected dividends for the closed-end fund become smaller. Thus, Corollary 2 indicates that there exists a negative relationship between the expected dividends of a closed-end fund and the magnitude of its discount, which is consistent with the findings by Pontiff (1996) and Johnson et al. (2006). However, the impact of the interest rate r on the magnitude of the discount is more complicated since r shows up not only in the denominator outside the brackets in Equation (8) but also in the two items inside the brackets. We will study the impact of r via simulation.

3. SIMULATION

3.1. Parameter Calibration

We choose 4.21%, the median of monthly 10-year Treasury constant maturity nominal yields from January 2000 to

January 2020 as the typical value of the discount rate r for our simulation.

Next, we estimate the typical values of δ_a and δ_b. Without loss of generality, we assume that $\delta_a = \delta_b$ (our main results are not altered even if δ_a is not equal to δ_b) According to Mckinsey Corporate Performance Analytics, the average annualized 5-year volatility for the S and P 500 index over the past 50 years is about 15%[3]. Meanwhile, Buraschi et al., (2009) find that the average implied volatility of the *S and P* 100 index option is about 19.2% while the average implied volatility of single-stock options on S&P 100 constituents is about 32.7%. Thus, we choose 0.3 (30%) as the typical values of δ_a and δ_b if we assume that the volatility of the stock price is commensurate with that of expected dividends. Applying a constant dividends stock valuation model, a bullish investor would value stock a (or b) as 1/4.21%=$23.75 per share, and the corresponding value for a bearish investor would be (1–0.3)/4.21%=$16.63 per share.

There are two ways to estimate the typical values of the selling rate and buying rate, α and β. Without bias, we assume that α is comparable with β. Based on the data from Morningstar, the average turnover rate for domestic funds is about 0.63 as of 2019[4]. Furthermore, if we can find out the average holding period of stocks, α and β can also be estimated by the inverse of the average holding period. According to a white paper published by MFS Investment Management Canada Limited, the average holding period for stocks traded on the NYSE is about 1.92 years although this number has been significantly shortened recently[5]. Thus, the corresponding trading rates would be 1/1.92=0.52. Combining the above two approaches, we choose the average of 0.63 and 0.52 as the typical values of α and β, i.e., $\alpha \sim \beta = (0.63+0.52)/2 \approx 0.58$.

In addition, we need to calibrate the two jumping rates, π_u and π_d, which characterize the investor sentiment in the stock market. Again, we assume that π_u is comparable with π_d as well in order to avoid any bias. Moreover, as a starting point, we assume that on average investors change their attitude to the stock every half a year. In other words, the two jumping rates, π_u and π_d equal 2. Furthermore, since there exists an obstacle for people to turn their desire into action, the measure for the investors' willingness to trade is always larger than that for the investors' ability to trade. Our calibrated values for the jumping rates and the selling/buying rates fall into this relationship (i.e., $\pi_u = \pi_d = 2 > 0.58 = \alpha = \beta$).

Lastly, we assume that the corresponding parameters for the closed-end fund and component stocks are all equal initially. In this way, there will be no discount or premium for the closed-end fund. However, a small perturbation of the values of parameters would cause the existence of either a discount or premium for the closed-end fund then. Table 1 summarizes the key parameters and the typical values used in our simulation.

3 https://www.mckinsey.com/business-functions/strategy-and-corporate-finance/our-insights/the-long-and-the-short-of-stock-market-volatility

4 https://www.investopedia.com/articles/mutualfund/09/mutual-fund-turnover-rate.asp

5 https://topforeignstocks.com/2017/10/01/average-stock-holding-period-on-nyse-1929-to-2016/

3.2. Simulation Results

In this session, we will study the impacts of selected parameters on the existence of a discount or premium of the closed-end fund and on its magnitude in order to evaluate the model's ability to reproduce the size of discounts/premiums under various scenarios.

3.2.1. The impact of the direction factor α_c

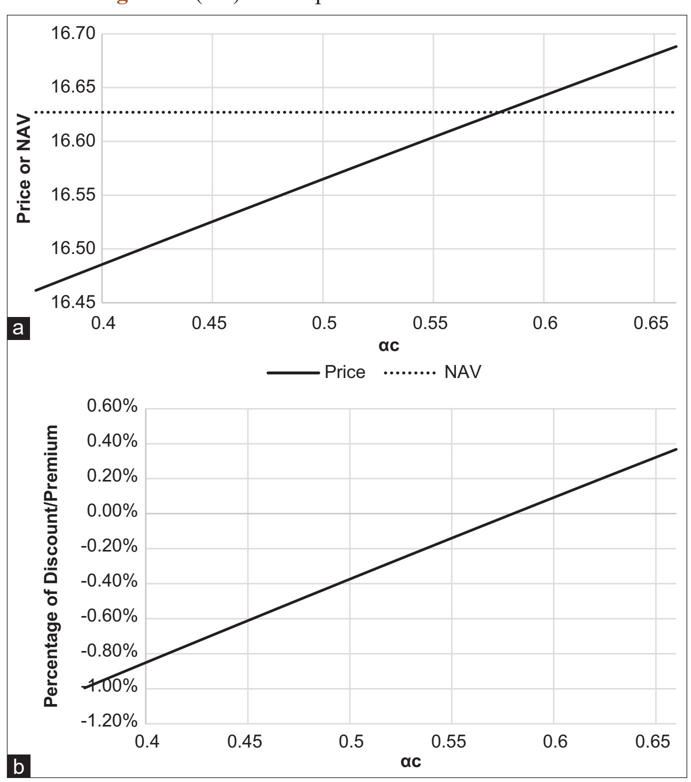
Figure 3 (a-b) shows the impact of the selling rate for the closed-end fund (α_c), one direction factor, on its discount or premium when the values of the other parameters are chosen as in Table I. The impact of the buying rate (β_c) can be easily analyzed accordingly.

In Figure 3 (a), when the selling rate for the closed-end fund α_c increases from 0.37 to 0.66, the share price of the closed-end fund P_c rises from $16.46 to $16.69 (the solid curve) while its net asset value is fixed at $16.63 per share (the dotted curve). Thus, the discount/premium defined as $\frac{P_c - NAV}{NAV}$ changes from −1.00% (discount) to +0.36% (premium), which is shown in Figure 3 (b).

Table 1: Parameter calibration

Name	Notation	Typical value
The discount rate	r	4.21%
The expected dividends loss for a bearish investor	δ_a (δ_b)	0.3
The selling rate during each period	α (α_c)	0.58
The buying rate during each period	β (β_c)	0.58
The jumping-up rate from a bearish investor to a bullish investor	π_u ($\pi_{u,c}$)	2
The jumping-down rate from a bullish investor to a bearish investor	π_d ($\pi_{d,c}$)	2

Figure 3: (a-b) The impact of the direction factor αc



Specifically, when $\alpha_c = \alpha = 0.58$, there is no discount or premium for the closed-end fund.

In other words, *ceteris, paribus,* when the selling rate for the closed-end fund α_c is less than the selling rate for stock a and b, P_c is less than NAV. Thus, there exists a discount for the closed-end fund; when the selling rate for the closed-end fund α_c is larger than the selling rate for stock a and b, P_c is larger than NAV. Thus, there exists a premium for the closed-end fund, which is consistent with Corollary 1.

3.2.2. The impact of the direction factor $\pi_{u,c}$

Figure 4 (a-b) represents the impact of the jumping-up rate from a bearish investor to a bullish investor for the closed-end fund ($\pi_{u,c}$), another direction factor, on its discount or premium when the values of the other parameters are chosen as in Table 1. Again, the impact of the jumping-down rate from a bullish investor to a bearish investor for the closed-end fund ($\pi_{d,c}$), can be easily analyzed and is thus not shown here.

In Figure 4 (a), when $\pi_{u,c}$ increases from 0.95 to 2.40, the share price of the closed-end fund P_c rises from \$14.53 to \$17.19 (the solid curve) while its net asset value is always fixed at \$16.63 per share (the dotted curve) as before. Correspondingly, the closed-end fund initially has a discount of 12.63%. However, with the increase in the jumping-up rate $\pi_{u,c}$, the discount is gradually inverted into a premium, which is illustrated in Figure 4 (b). When $\pi_{u,c} = \pi_u = 2$, there is no discount or premium for the closed-end fund.

In summary, *ceteris, paribus,* when the jumping-up rate from a bearish investor to a bullish investor for the closed-end fund ($\pi_{u,c}$) is less than the corresponding rate for stock a and b, there exists a discount for the closed-end fund; *vice versa*, there exists a premium.

Moreover, compared to the effect of the selling rate for the closed-end fund (α_c) discussed previously (the discount/premium changes from −1.00% to +0.36%), the impact of the jumping-down rate from a bullish investor to a bearish investor for the closed-end fund ($\pi_{d,c}$) (the discount/premium changes from −12.63% to +3.37%) seems more profound. Since the jumping rates between a bearish investor and a bullish investor in our model represents a direct measure of investor sentiment(i.e., the investors' the willingness to trade), this simulation results imply that the fluctuations of investor sentiment in the partially integrated markets could be the most dominating factor to explain the existence of a discount or premium.

3.2.3. The impact of the modulating factor ($\delta_a + \delta_b$)

Figure 5 (a-b) illustrates the impact of one modulating factor ($\delta_a + \delta_b$), the loss of dividends for the closed-end fund expected by a bearish investor, on the magnitude of the closed-end fund discount. To incur a discount, except ($\delta_a + \delta_b$), we let the values of the other parameters for stock a and b fixed as in Table I (i.e., $\alpha=\beta=0.58$, $\pi_u=\pi_d=2$). The corresponding values for the closed-end fund are chosen according to Corollary 1 (i.e., $\alpha_c=0.54$, $\beta_c=0.62$, $\pi_{u,c}=1.8$, $\pi_{d,c}=2.2$).

In Figure 5 (a), when ($\delta_a + \delta_b$) increases from 0.30 to 0.88, equivalent to a decrease in expected dividends for the closed-end fund from 1.70 (=2-0.30) to 1.12 (=2-0.88), the share price of the closed-end fund P_c is reduced from \$19.85 to \$12.31 (the solid curve) and its net asset value is reduced from \$20.19 to \$13.30

Figure 4: (a-b) The impact of the direction factor πu, c

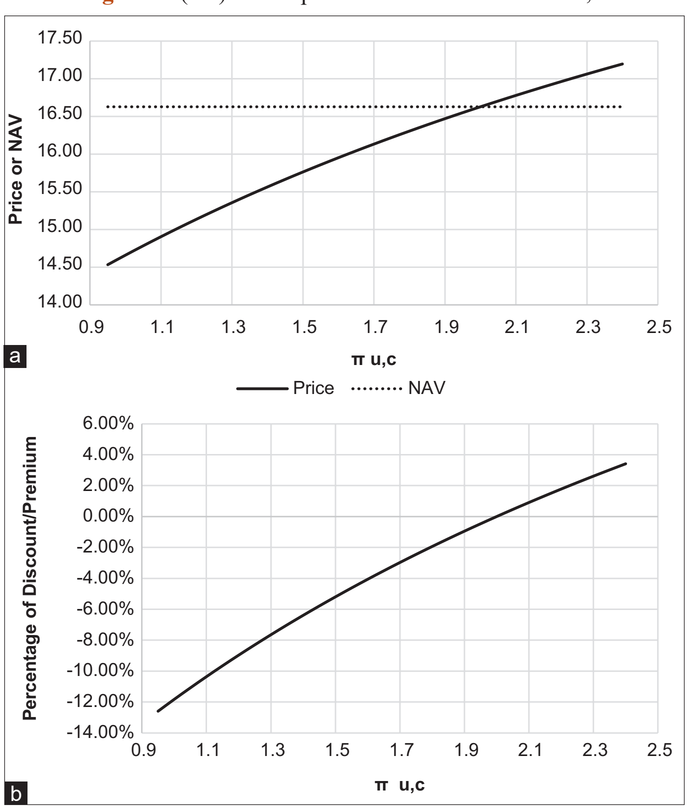
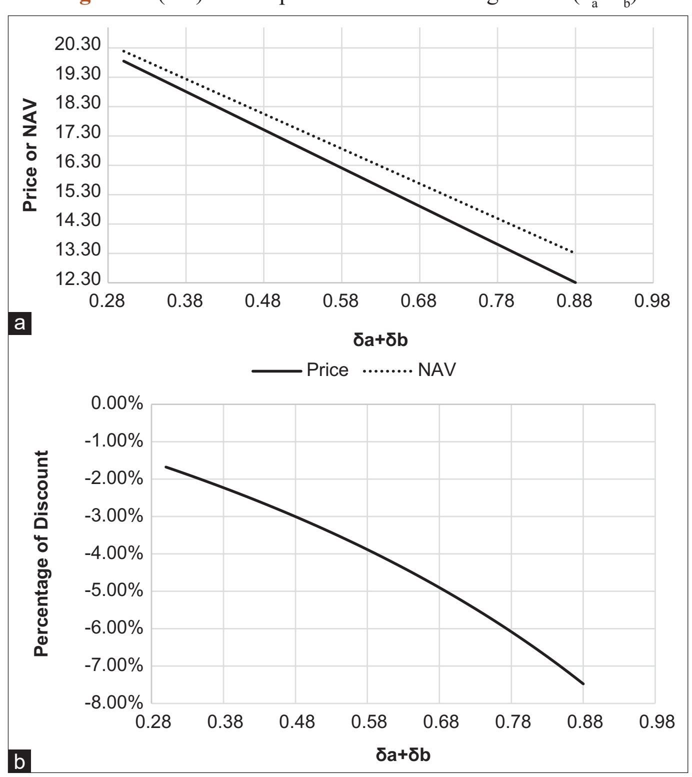


Figure 5: (a-b) The Impact of the Modulating Factor (δ_a+δ_b)



(the dotted curve) as well. However, the net effect is that with the decrease in expected dividends for the closed-end fund, its discount rises from 1.68% to 7.44% shown in Figure 5 (b).

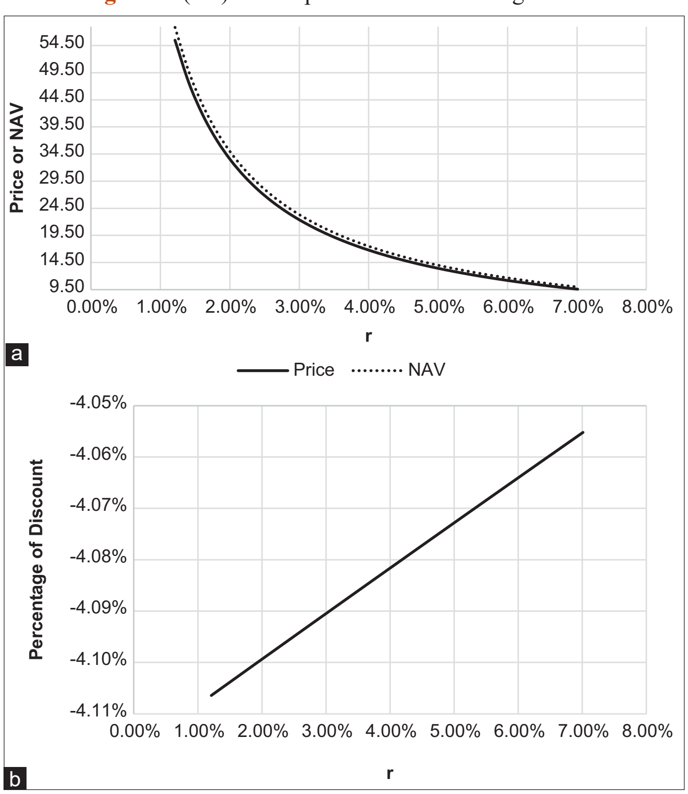
3.2.4. The impact of the modulating factor r

Another important modulating factor is the interest rate r. However, according to Corollary 2, its impact on the magnitude of the discount is more complicated. Only considering r shown in the denominator outside the brackets in Equation (8), we could conjecture that the closed-end fund discount measured in dollars is negatively related to the interest rate r. However, if we consider the effect of r inside of the brackets in Equation (8) and calculate the discount in percentage, the net influence of r cannot be resolved purely based on theory but by simulation.

Figure 6 (a-b) shows the impact of r on the magnitude of the closed-end fund discount when the values of the other parameters for stock a and b are fixed as in Table I (i.e., $\alpha=\beta=0.58$, $\pi_u=\pi_d=2$) and the corresponding values for the closed-end fund are chosen according to Corollary 1 (i.e., $\alpha_c=0.54$, $\beta_c=0.62$, $\pi_{u,c}=1.8$, $\pi_{d,c}=2.2$) again.

In Figure 6 (a), when r increases from 1.21% to 7.01%, the share price of the closed-end fund P_c decreases from \$55.48 to \$9.58 (the solid curve) and its net asset value decreases from \$57.85 to \$9.99 (the dotted curve) as well. Moreover, according to Figure 6 (b), with the increase in r, the closed-end fund discount only decreases from 4.11% to 4.06% under the current parameters. However, compared to the strong influence of expected dividends for the closed-end fund, the negative relationship between the interest rate and the magnitude of the discount is not salient.

Figure 6: (a-b) The Impact of the Modulating Factor r



4. CONCLUSIONS

In this paper, we set up a dynamic capital mobility model to rationalize the existence of a discount for a closed-end fund's price with respect to its net asset value (NAV). The underlying assumption of our model is that due to some constraint arbitrageurs cannot mobilize their capital to fully equalize the key dynamic parameters among partially integrated markets. Although the existence of a discount for a closed-end fund is more prevalent in the current financial market, our model also suggests the possibility of a premium, all depending on the relative magnitude of the key parameters for the markets of the closed-end fund and component stocks.

We make a difference between the direction factors, for instance, the selling rate and the jumping rate between a bearish investor and a bullish investor, which determine the existence of a discount or premium; and the modulating factors, for instance, the expected dividends and the interest rate, which enhance or weaken the above effect.

Since closed-end funds tend to be more owned by individual investors who are less likely to be active traders due to investor inertia, and investor enthusiasm for stocks is usually higher than that for a closed-end fund, the key parameters in the market of the closed-end fund and the markets of component stocks are different in two main aspects: (1) the investors' ability to trade and (2) the investors' willingness to trade. Thus, the selling rate for stocks which measures the investors' ability to sell is typically larger than that for a closed-end fund, and the jumping-up rate from a bearish investor to a bullish investor for stocks which reflects the investors' willingness to trade is mostly larger than that for a closed-end fund. Combining the above two effects, the aggregated price of component stocks will be more likely higher than the price of the closed-end fund, leading to a prevalence of a discount for closed-end funds. Our simulation results show that the second effect seems more profound than the first one, thus indicating that investor sentiment could be the most dominating factor to interpret the existence of a discount or premium.

Our model also shows that there exists a negative relationship between the size of expected dividends for a closed-end fund and the magnitude of its discount. In other words, the larger the size of the expected dividends for the closed-end fund, the less the magnitude of its discount. However, the modulating impact of the interest rate r can only be answered by simulation. Our simulation results imply that there exists a rather weakly negative relationship between the interest rate and the discount under the current parameters.

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APPENDIX

APPENDIX A: SYMBOLS AND NOTATIONS

H_0: Bullish investor without one share of the stock (buyer)

H_1: Bullish investor with one share of the stock

L_0: Bearish investor without one share of the stock

L_1: Bearish investor with one share of the stock (seller)

V_{H0}: The value of a bullish investor without one share of the stock

V_{H1}: The value of a bullish investor with one share of the stock

V_{L0}: The value of a bearish investor without one share of the stock

V_{L1}: The value of a bearish investor with one share of the stock

Subscript a: Represents stock a

Subscript b: Represents stock b

Subscript c: Represents the closed-end fund consisting of stock a and stock b

r: The discount rate

P: The transaction price of one share of an individual stock

P_a: The transaction price of one share of stock a

P_b: The transaction price of one share of stock b

P_c: The transaction price of one share of the closed-end fund

$P_a + P_b$: The net asset value (NAV) of the closed-end fund if each share of the closed-end fund corresponds to one share of stock a and one share of stock b.

δ: When holding one share of stock, a bearish investor expects to receive (1- δ) units of dividends during each period while a bullish investor expects to receive 1 unit of dividends during each period, both measured in consumption goods.

α: The selling rate during each period

β: The buying rate during each period

π_u: The jumping-up rate from a bearish investor to a bullish investor

π_d: The jumping-down rate from a bullish investor to a bearish investor

APPENDIX B: PROOFS OF PROPOSITIONS

Proposition 1

Subtract Equation (4) from Equation (1), we obtain:

$r(V_{L1} - V_{L0}) = 1-\delta + \pi_u(V_{H1} - V_{L1} - V_{H0} + V_{L0}) + \alpha(P + V_{L0} - V_{L1})$

Replace P by $(\Delta V_L + \Delta V_H)/2$,

$r\,\Delta V_L = 1-\delta + \pi_u(\Delta V_H - \Delta V_L) + \alpha[(\Delta V_L + \Delta V_H)/2 - \Delta V_L]$

$r\,\Delta V_L = 1-\delta + \pi_u \Delta V_H - \pi_u \Delta V_L - \alpha \Delta V_L/2 + \alpha \Delta V_H/2$

$(r+\pi_u+\alpha/2)\,\Delta V_L = 1-\delta + (\pi_u + \alpha/2)\Delta V_H$ (B-1)

Subtract Equation (2) from Equation (3), we obtain:

$r(V_{H1} - V_{H0}) = 1 + \pi_d(V_{L1} - V_{H1} - V_{L0} + V_{H0}) - \beta(V_{H1} - V_{H0} - P)$

Replace P by $(\Delta V_L + \Delta V_H)/2$,

$r\,\Delta V_H = 1 + \pi_d(\Delta V_L - \Delta V_H) - \beta[\Delta V_H - (\Delta V_L + \Delta V_H)/2]$

$r\,\Delta V_H = 1 + \pi_d \Delta V_L - \pi_d \Delta V_H - \beta \Delta V_H + \beta \Delta V_L/2 + \beta \Delta V_H/2$

$(r + \pi_d + \beta/2)\,\Delta V_H = 1 + (\pi_d + \beta/2)\Delta V_L$ (B-2)

Combine Equation (B-1) and (B-2) to solve for ΔV_H and ΔV_L:

$$\Delta V_L = \frac{1}{r} - \frac{\delta}{r}\,\frac{(r + \pi_d + \frac{\beta}{2})}{(r + \pi_u + \pi_d + \frac{\beta}{2} + \frac{\alpha}{2})} \qquad \text{(B-3)}$$

$$\Delta V_H = \frac{1}{r} - \frac{\delta}{r}\,\frac{(\pi_d + \frac{\beta}{2})}{(r + \pi_u + \pi_d + \frac{\beta}{2} + \frac{\alpha}{2})} \qquad \text{(B-4)}$$

Then, the transaction price of one share of stock P in market equilibrium is:

$P = (\Delta V_L + \Delta V_H)/2$

$$\frac{1}{r} - \frac{\delta}{r}\,\frac{(\frac{r}{2} + \pi_d + \frac{\beta}{2})}{(r + \pi_u + \pi_d + \frac{\beta}{2} + \frac{\alpha}{2})} \qquad (6)$$

Proposition 2

Substituting $\pi_{d,a}$ and $\pi_{d,b}$ by π_d and substituting $\pi_{u,a}$ and $\pi_{u,b}$ by π_u in Equation (7), let $\delta_c = \delta_a + \delta_b$ then collect items. The discount/premium of the closed-end fund measured in dollars is expressed:

$$P_c - NAV = P_c - (P_a + P_b)$$

$$= \frac{\delta_a + \delta_b}{r}\left[\frac{\left(\frac{r}{2} + \pi_d + \frac{\beta}{2}\right)}{\left(r + \pi_u + \pi_d + \frac{\beta}{2} + \frac{\alpha}{2}\right)} - \frac{\left(\frac{r}{2} + \pi_{d,c} + \frac{\beta_c}{2}\right)}{\left(r + \pi_{u,c} + \pi_{d,c} + \frac{\beta_c}{2} + \frac{\alpha_c}{2}\right)}\right. \qquad (8)$$

Corollary 1

When all the parameters in Equation (8) except α are equal for the closed-end fund and its component stocks, P_c<NAV if $\alpha > \alpha_c$;

When all the parameters in Equation (8) except β are equal for the closed-end fund and its components, P_c<NAV if $\beta < \beta_c$;

When all the parameters in Equation (8) except π_u are equal for the closed-end fund and its components, P_c<NAV if $\pi_u > \pi_{u,c}$;

When all the parameters in Equation (8) except π_d are equal for the closed-end fund and its components, P_c<NAV if $\pi_d < \pi_{d,c}$.

Thus, when the four conditions ($\alpha > \alpha_c$, $\beta < \beta_c$, $\pi_u > \pi_{u,c}$, and $\pi_d < \pi_{d,c}$) are all satisfied simultaneously, then P_c<NAV, i.e., there exists a discount for the closed-end fund.

Corollary 2

Supposing that parameters in Equation (8) can produce a discount for the closed-end fund (i.e., P_c-NAV<0), observe the coefficient outside the brackets, $\frac{\delta_a + \delta_b}{r}$. Then, the larger the size of $(\delta_a + \delta_b)$, the larger the absolute value of (P_c-NAV), which implies that the discount of the closed-end fund is more severe. However, while a higher value of r leads to a decrease in the coefficient $\frac{\delta_a + \delta_b}{r}$, the impact of r on the two items inside the brackets is not straightforward without empirical data.