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A Generalized Interest Rates Model with Scaling

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ABSTRACT

The article introduces scaling and generalizes the Taylor (1993) interest rate rule from four terms to seven terms. The three additional terms are the deviation in money supply, the deviation in money velocity, and the deviation in unemployment rate. The four original terms are the inflation rate, the equilibrium real interest rate, the deviation in inflation rate, and the deviation in real GDP (Gross Domestic Product). The weights for the seven terms are estimated via the monthly January 1, 1959-March 31, 2022 US data. All the seven combinations of the Taylor (1993) rule, the Quantity Equation (Friedman, 1970), and the Phillips (1958) curve with scaling give substantially better results than both the Taylor (1993; 1999) rules without scaling. The Phillips (1958) curve is best when choosing only one rule with scaling. Combining the Taylor (1993) rule and the Phillips (1958) curve is best when choosing between two rules with scaling.

Keywords: Monetary Policy, Taylor Rules, Phillips Curve, Interest Rate, Inflation Rate, Money Supply, Money Velocity, Unemployment Rate
JEL Classifications: C6, E24, E50, E47, E52, E58

1. INTRODUCTION

1.1. Background

Interest rates have been a hot topic in academic research for a long time. Central banks apply discretion and various rules to adjust interest rates to ensure economic stability and monetary liquidity. The best known policy rule is the Taylor (1993) rule. It recommends that central banks adjust interest rates in response to four terms, i.e. the inflation rate, long term equilibrium real interest rate, deviation in inflation rate, and the deviation in real GDP (Gross Domestic Product). The Taylor (1993) rule has received substantial attention in academic research. Various interest rate rules have emerged after the Taylor (1993) rule, e.g. the Taylor (1999) rule, balanced-approach rule, inertial Taylor rule, effective lower bound-adjusted rule, first-difference rule, etc. (Erceg et al., 2012). The Taylor (1993) rule has four terms, i.e. the inflation rate, the equilibrium real interest rate, the deviation in inflation rate, and the deviation in real GDP. Taylor (1993) assigns equal 0.5 weight to both the deviation in real GDP and the deviation in inflation rate. Subsequently, in his Taylor (1999) rule, he increases the weight for the deviation in real GDP to one. Perhaps surprisingly, both Taylor (1993; 1999) rules assign default weight one to the inflation rate and the equilibrium real interest rate.

1.2. Contribution

Building upon this background, it seems interesting to explore additional phenomena beyond Taylor's (1993; 1999) four terms, and assess how the terms should be scaled relative to each other. The article investigates and generalizes the Taylor (1993) rule from four terms to seven terms on the right hand side to determine the interest rate on the left hand side. The three additional terms are two terms from the Quantity Equation (Friedman, 1970), i.e. the money supply and money velocity, and one term from the Phillips (1958) curve, i.e. the unemployment rate. The article estimates weights for the seven terms, which amounts to scaling them relative to each other. To our best knowledge, this article is the first to explore the scaling issue for Taylor (1993; 1999) rules or generalizations of such rules. The article adopts monthly US January 1, 1959-March 31, 2022 US data in the empirical analysis. The article uses the least squares method to estimate the optimal weights for the seven terms.

1.3. Literature

The Taylor (1993) rule suggests an equal 0.5 weight for the deviation in inflation rate and the deviation in real GDP. The Taylor (1999) rule keeps the 0.5 weight for the deviation in inflation rate, but increases the weight assigned to the deviation in real GDP to one. Several monetary rules are based on the Taylor (1993) rule, e.g. the effective lower bound-adjusted rule (Reifschneider and Williams, 2000). It suggests that the interest rate cannot be lower than the so-called effective lower bound. The first difference rule (Orphanides, 2003) connects the current interest rate to its previous value. The inertial rule (Bullard, 2017; Kliesen, 2019) lowers the interest rate's volatility over time, and points out that the policymaker adjusts the interest rate gradually. Taylor and Williams (2010) provide a comprehensive review of interest rate policy rules.

The Quantity Equation (Friedman, 1970) connects the money supply, money velocity, price level (or inflation rate), and the real GDP. Money supply is widely assumed to impact interest rates. For example, Friedman (1961) suggests that the money supply has a negative effect on the interest rate. Money velocity also relates to the interest rate. Taylor (1999, p. 322) says that "we know that velocity depends on the interest rate and on real output or income." Keynes et al. (1971) suggest an inverse relationship between the money velocity and the money supply. In addition, money velocity may also impact the interest rate via the inflation rate (Mendizabal, 2006). But both money supply and money velocity are absent in the Taylor (1993, 1999) rules. Prag (1994) suggests an inverse relationship between the interest rate and the unemployment rate. The unemployment rate is also absent in the Taylor (1993, 1999) rules.

The literature compares the interest rate rules with other policy rules, e.g. money supply rules (Ascari and Ropele, 2013; Auray and Fève, 2003; Schabert, 2005; Srinivasan, 2000), McCallum rule Razzak (2003), Friedman rule (Srinivasan, 2000), etc. The literature also links monetary policy to macroeconomics (Clarida et al., 2000; Schabert, 2009; Wijngaard and Van Hee, 2021; Woodford, 2001), to the Phillips (1958) curve (Wang and Hausken, 2022a), adopts the Taylor (1993) rule to design decision models (Wang and Hausken, 2022b), and builds dynamic stochastic general equilibrium models (Ferrari Minesso et al., 2022; Oh and Zhang, 2020).

1.4. Article Organization

Section 2 presents the model. Section 3 analyzes the model with data sources, parameter estimation, and illustrations. Section 4 concludes.

2. THE MODEL

Appendix A shows the nomenclature. This article generalizes the Taylor (1993) rule. First, it introduces three additional terms, i.e. money supply m_t, $m_t>0$, and money velocity v_t, $v_t>0$, as presented in the Quantity Equation (Friedman, 1970), and unemployment rate u_t, $u_t \geq 0$ as presented in the Phillips (1958) curve. Second, it incorporates scaling for the seven terms, thus making the weights assigned to the seven terms comparable. Thus the interest rate i_t at time t is given by

$$i_t = a_{pi}s_{pi}\pi_t + a_r s_r r_t^* + a_\pi s_\pi\left(\pi_t - \pi_t^*\right) + a_y s_y Log\left(\frac{y_t}{\bar{y}_t}\right)$$

$$+a_m s_m Log\left(\frac{m_t}{\bar{m}_t}\right) + a_v s_v Log\left(\frac{v_t}{\bar{v}_t}\right) + a_u s_u\left(\bar{u}_t - u_t\right),$$

$$s_{pi} \equiv \frac{1}{\left(P_{pi}\sum_{h=1}^{P_{pi}}\pi_h + N_{pi}\left|\sum_{k=1}^{N_{pi}}\pi_k\right|\right)/\left(P_{pi}+N_{pi}\right)},$$

$$s_r \equiv \frac{1}{\left(P_r\sum_{h=1}^{P_r}r_h^* + N_r\left|\sum_{k=1}^{N_r}r_k^*\right|\right)/\left(P_r+N_r\right)},$$

$$s_\pi \equiv \frac{1}{\left(P_\pi\sum_{h=1}^{P_\pi}\left(\pi_h-\pi_h^*\right) + N_\pi\left|\sum_{k=1}^{N_\pi}\left(\pi_k-\pi_k^*\right)\right|\right)/\left(P_\pi+N_\pi\right)},$$

$$s_y \equiv \frac{1}{\left(P_y\sum_{h=1}^{P_y}Log\left(\frac{y_h}{\bar{y}_h}\right) + N_y\left|\sum_{k=1}^{N_y}Log\left(\frac{y_k}{\bar{y}_k}\right)\right|\right)/\left(P_y+N_y\right)},$$

$$(1)$$

$$s_m \equiv \frac{1}{\left(P_m\sum_{h=1}^{P_m}Log\left(\frac{m_h}{\bar{m}_h}\right) + N_m\left|\sum_{k=1}^{N_m}Log\left(\frac{m_k}{\bar{m}_k}\right)\right|\right)/\left(P_m+N_m\right)},$$

$$s_v \equiv \frac{1}{\left(P_v\sum_{h=1}^{P_v}Log\left(\frac{v_h}{\bar{v}_h}\right) + N_v\left|\sum_{k=1}^{N_v}Log\left(\frac{v_k}{\bar{v}_k}\right)\right|\right)/\left(P_v+N_v\right)},$$

$$s_u \equiv \frac{1}{\left(P_u\sum_{h=1}^{P_u}\left(\bar{u}_h-u_h\right) + N_u\left|\sum_{k=1}^{N_u}\left(\bar{u}_k-u_k\right)\right|\right)/\left(P_u+N_u\right)}$$

where $i_t \in R$, R is the set of all real numbers, $t \geq 0$, r_t^* is the equilibrium real interest rate, y_t is the real GDP, $y_t \geq 0$, $\bar{y}_t$ is the potential real GDP that can be sustained in the long run, $\bar{y}_t \geq 0$. The right hand side of (1) contains the four original terms in the Taylor (1993) rule, i.e π_t, r_t^*, $\pi_t - \pi_t^*$ and $Log\left(\frac{y_t}{\bar{y}_t}\right)$, where $\pi_t - \pi_t^*$ is the deviation in inflation rate, $Log\left(\frac{y_t}{\bar{y}_t}\right)$ is the deviation in real GDP. The three new terms in (1) are the deviation $Log\left(\frac{m_t}{\bar{m}_t}\right)$ in money supply, the deviation $Log\left(\frac{v_t}{\bar{v}_t}\right)$ in money velocity, and the deviation $\bar{u}_t - u_t$ in unemployment rate, where, Log is the logarithm with a base of 10, m_t is the money supply, $m_t \geq 0$, $\bar{m}_t$ is the potential money supply, $\bar{m}_t \geq 0$, v_t is the money velocity $v_t \geq 0$, $\bar{v}_t$ is the potential money velocity, $\bar{v}_t \geq 0$, $\bar{u}_t$ is the natural unemployment rate, $\bar{u}_t \geq 0$, and u_t is the unemployment rate, $u_t \geq 0$.

In (1), s_j, $j=pi,r,\pi,y,m,v,u$ are the scaling parameters for the seven terms. These are the inflation rate π_t, the equilibrium real interest rate r_t^*, the deviation $\pi_t - \pi_t^*$ in inflation rate, the deviation

$Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP, the deviation $Log\left(\frac{m_t}{\bar{m}_t}\right)$ in money supply, the deviation $Log\left(\frac{v_t}{\bar{v}_t}\right)$ in money velocity, and the deviation $\bar{u}_t - u_t$ in unemployment rate, respectively, where P_j, $j=pi,r,\pi,y,m,v,u$ specifies the number of nonnegative numbers in the data for term j, and N_j, $j=pi,r,\pi,y,m,v,u$ specifies the number of negative numbers in the data for term j. Hence N_j is multiplied by the absolute value of the sum of the negative data points for term j in (1). The sum $P_j+N_j=759$ specifies the number of data points for the period January 1, 1959-March 31, 2022. We introduce P_j and N_j to ensure proper and intuitive scaling, since data points may be negative or positive. The counting parameters h and k are associated with P_j and N_j, respectively, to run through the $P_j+N_j=759$ data points. The seven parameters $a_{pi}, a_r, a_\pi, a_y, a_m, a_v, a_u$ are the weights assigned to the seven terms, which can be positive or nonpositive. If the weight is positive, it means that the corresponding term positively impacts the interest rate i_t. If the weight is negative, it means that the corresponding term negatively impacts the interest rate i_t.

The four terms in (1), i.e. the inflation rate π_t, the equilibrium real interest rate r_t^*, the deviation $\pi_t - \pi_t^*$ in inflation rate and the deviation $Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP, are originally included in the Taylor (1993, 1999) rules. The Taylor (1993, 1999) rules assign default weight one to both the inflation rate π_t and the equilibrium real interest rate r_t^*.

The first new term is the deviation $Log\left(\frac{m_t}{\bar{m}_t}\right)$ in money supply. Thus, the two variables the money supply m_t and the potential money supply $\bar{m}_t$ are introduced. We adopt the standard Hodrick and Prescott (1997) filter to estimate the potential money supply $\bar{m}_t$. The method is widely used in macroeconomics to investigate the potential GDP, especially in real business cycle theory (Furceri and Mourougane, 2012). The interest rate i_t is the price of the money supply m_t applying supply and demand considerations. As Friedman (1961) suggests, money supply m_t has a negative effect on the interest rate i_t. Conrad (2021) also points out that the interest rate i_t decreases when the money supply m_t increases. Nevertheless, central banks may choose to increase the interest rate i_t to prevent savers' extensive withdrawals when the money supply m_t increases. This is consistent with Ascari and Ropele (2013). They suggest a positive relationship between the money supply m_t and the interest rate i_t.

The second new term is the deviation $Log\left(\frac{v_t}{\bar{v}_t}\right)$ in money velocity. The money velocity v_t and the potential money velocity $\bar{v}_t$ are introduced. The two variables are present in the Quantity Equation (Friedman, 1970). The money velocity v_t is defined as the ratio of nominal GDP to the money supply (Federal Reserve Bank of St. Louis, 2022). The potential money velocity $\bar{v}_t$ is defined as the ratio of nominal potential GDP to the potential money supply. The money velocity v_t is widely accepted to have a positive impact on the inflation rate π_t (Mendizabal, 2006). This is consistent with Taylor (1993, 1999) assuming positive correlation between the inflation rate π_t and the interest rate i_t. Thus, the money velocity v_t may affect the interest rate i_t positively.

The third new term is the deviation $\bar{u}_t - u_t$ in the unemployment rate. The unemployment rate u_t is present in the short run Phillips (1958) curve. It shows an inverse relationship between the inflation rate π_t and the unemployment rate u_t over the short run. Taylor (1993) assumes a positive correlation between the inflation rate π_t and the interest rate i_t. Hence, the unemployment rate u_t may impact the interest rate i_t negatively. Summing up, as specified in (1), the seven weights of the seven terms scale these terms relative to each other, and scale them overall relative to the interest rate i_t on the left hand side.

3. ANALYZING THE MODEL

3.1. Data Sources

This article uses the monthly US data. The data range is from January 1, 1959 to March 31, 2022, collected and estimated from the following sources. We estimate the real GDP y_t and the real potential GDP $\bar{y}_t$ from the US Bureau of Economic Analysis (2022) and the US Congressional Budget Office (2022b), respectively. We apply the quadratic interpolation method to convert quarterly data to monthly data for the real GDP y_t and the real potential GDP $\bar{y}_t$. We estimate the M2 money supply m_t and the money velocity v_t from the Board of Governors of the Federal Reserve System (US) (2022b), and the Federal Reserve Bank of St. Louis (2022), respectively. The unemployment rate u_t and the natural unemployment rate $\bar{u}_t$ are estimated from the US Bureau of Labor Statistics (2022b) and the US Congressional Budget Office (2022a), respectively. Again, we adopt the quadratic interpolation method to convert quarterly data to monthly data for the natural unemployment rate $\bar{u}_t$. The inflation rate π_t and the empirical interest rate i_t are derived from the US Bureau of Labor Statistics (2022a), and the Board of Governors of the Federal Reserve System (US) (2022a), respectively. The target inflation rate π_t^* is from several sources. We set the target inflation rate $\pi_t^*=1.5\%$ from January 1, 2000 to December 30, 2007 inspired by Shapiro and Wilson (2019). For the remaining January 1, 1959-March 31, 2022 time periods, we use the common $\pi_t^*=2\%$, as Taylor (1993) assumes for January 1, 1984 to September 31, 1992. Finally, we use the common equilibrium real interest rate $r_t^*=2\%$ from January 1, 1959 to March 31, 2022, as used by Taylor (1993) for January 1, 1984 to September 31, 1992, and consistent with the estimation of Kiley (2020).

3.2. Estimating the Parameters and Illustrating the Solutions

Table 1 shows the estimations of the seven parameter values $a_{pi}, a_r, a_\pi, a_y, a_m, a_v, a_u$ in (1), the sum S of the squared differences between the empirical interest rate i_t and the estimated interest rate i_t in (1), the number N of free choice variables for each estimation, and the specifics of each estimation.

Curve 1 assumes seven free choice variables, and represents the combination of the Taylor (1993) rule, the Quantity Equation

Table 1: Curve label, estimated parameter values $a_{pi},a_r,a_\pi,a_y,a_m,a_v,a_u$, the sum S of the squared differences between the empirical interest rate i_t and the estimated interest rate i_t in (1), the number N of free choice variables, and the estimation specifics

Curve	a_{pi}	a_r	a_π	a_y	a_m	a_v	a_u	S	N	Estimation specifics
1	66.72	–11.44	–16.84	–3.13	1.29	2.73	2.06	0.44567	7	Combination of the Taylor (1993) rule, the Quantity Equation (Friedman, 1970), and the Phillips (1958) curve with scaling, optimizing $a_{pi},a_r,a_\pi,a_y,a_m,a_v,a_u$
2	61.94	–9.00	–14.75	–1.12	1.25	2.98	0	0.45341	6	$a_u=0$, combination of the Taylor (1993) rule and the Quantity Equation (Friedman, 1970) with scaling, optimizing $a_{pi},a_r,a_\pi,a_y,a_m,a_v$
3	71.23	–12.69	–18.61	–0.73	0	0	2.19	0.45157	5	$a_m=a_v=0$, combination of the Taylor (1993) rule and the Phillips (1958) curve with scaling, optimizing a_{pi},a_r,a_y,a_π,a_u
4	25.72	11.32	0	0	0.41	0.94	0.92	0.45628	5	$a_\pi=a_y=0$, combination of the Quantity Equation (Friedman, 1970) and Phillips (1958) curve with scaling, optimizing a_{pi},a_r,a_y,a_π,a_u
5	66.14	–10.02	–16.34	1.64	0	0	0	0.46065	4	$a_m=a_v=a_u=0$, Taylor (1993) rule with scaling, optimizing a_{pi},a_r,a_y,a_π
6	25.82	10.77	0	0	0.77	1.88	0	0.45850	4	$a_\pi=a_y=a_u=0$, Quantity Equation (Friedman, 1970) with scaling, optimizing a_{pi},a_r,a_m,a_v
7	25.68	11.58	0	0	0	0	1.55	0.45780	3	$a_\pi=a_y=a_m=a_v=0$, Phillips (1958) curve with scaling, optimizing a_{pi},a_r,a_u
8	27.44	15.18	5.55	1.88	0	0	0	0.83070	0	Taylor (1993) rule
9	27.44	15.18	5.55	3.76	0	0	0	0.81949	0	Taylor (1999) rule
Average 1-7								0.45094	0	Average of curves 1-7
Average 1-9								0.47122	0	Average of curves 1-9

(Friedman, 1970), and the Phillips (1958) curve with scaling, where $a_{pi},a_r,a_\pi,a_y,a_m,a_v,a_u$ are optimized. With scaling no difference exists between the two Taylor (1993, 1999) rules, so we refer to the Taylor (1993) rule with scaling in general. That leads to the lowest sum of squares S=0.44567 in Table 1. The corresponding optimal weights are $a_{pi},a_r,a_\pi,a_y,a_m,a_v,a_u$ 66.72, –11.44, –16.84, –3.13, 1.29, 2.73 and 2.06, respectively. This indicates that the inflation rate π_t with a weight 66.72 is very explanatory to the interest rate i_t. Thereafter, in degree of explanatory power, follows the deviation $\pi_t-\pi_t^*$ In inflation rate with a negative weight –16.84, the equilibrium real interest rate π_t^* with a negative weight –11.44, and the deviation $Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP with a negative weight –3.13. The three new terms have lower weights. That is, the deviation $Log\left(\frac{v_t}{\bar{v}_t}\right)$ in money velocity has weight 2.73, the deviation $\bar{u}_t-u_t$ in unemployment rate has weight 2.06, and the deviation $Log\left(\frac{m_t}{\bar{m}_t}\right)$ in money supply has weight 1.29.

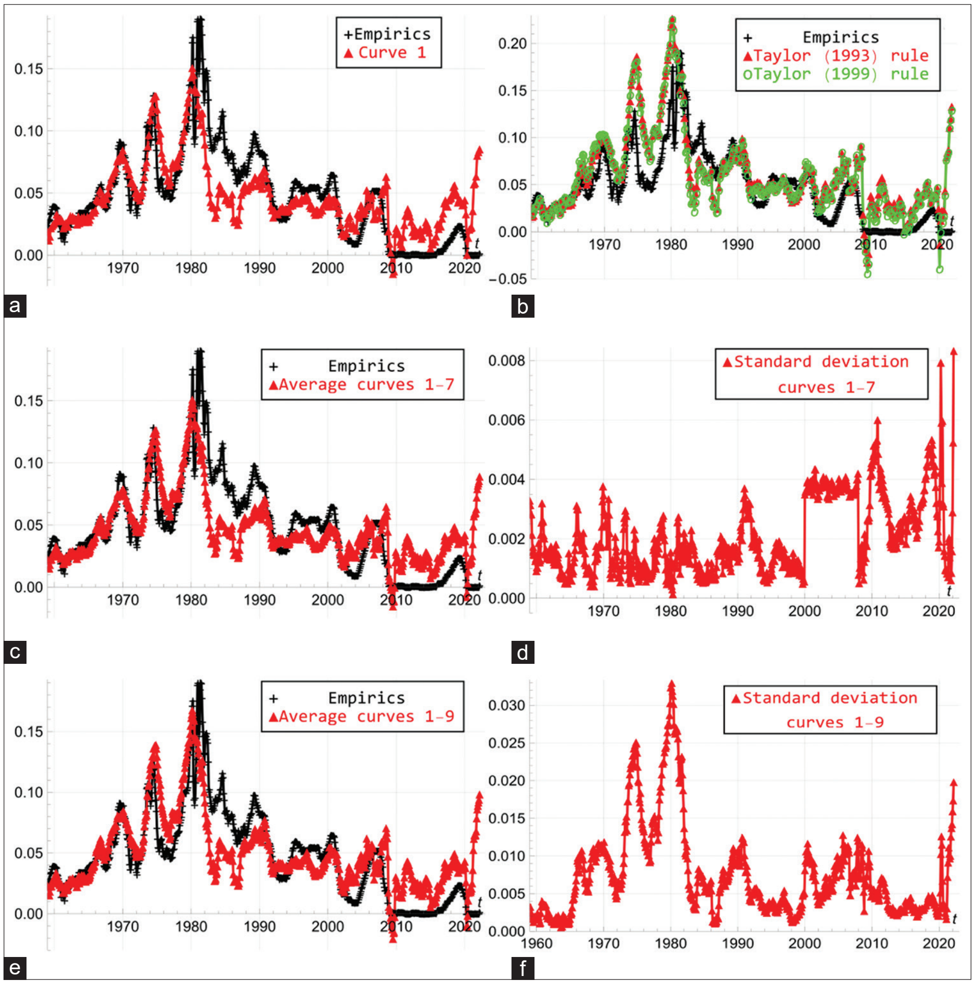
Curve 2 assumes six free choice variables, and represents the combination of the Taylor (1993) rule and the Quantity Equation (Friedman, 1970) with scaling, where $a_{pi},a_r,a_\pi,a_y,a_m,a_v$ are optimized assuming a_u = 0. That leads to a slightly higher sum of squares S=0.45341 compared to curve 1 in Table 1. The corresponding optimal weights $a_{pi},a_r,a_\pi,a_y,a_m,a_v$ are 61.94, –9.00, –14.75, –1.12, 1.25 and 2.98, respectively. Again, the inflation rate π_t has the highest weight 61.94 compared to the other five terms.

Curve 3 assumes five free choice variables, and represents the combination of the Taylor (1993) rule and the Phillips (1958) curve with scaling, where a_{pi},a_r,a_y,a_π,a_u are optimized assuming a_m=a_v=0. That leads to a sum of squares S=0.45157. The corresponding optimal weights a_{pi},a_r,a_y,a_π,a_u are 71.23, –12.69, –18.61, –0.73 and 2.19, respectively. Under the assumption a_m=a_v=0, the optimal weight assigned to the inflation rate π_t increases from 66.72 in curve 1 to 71.23 in curve 3. Meanwhile, the optimal weight assigned to the deviation $Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP increases from –3.13 in curve 1 to –0.73 in curve 3.

Curve 4 assumes five free choice variables, and represents the combination of the Quantity Equation (Friedman, 1970) and Phillips (1958) curve with scaling, where a_{pi},a_r,a_y,a_π,a_u are optimized assuming a_π=a_y=0. That causes a sum of squares S=0.45628. The corresponding optimal weights a_{pi},a_r,a_y,a_π,a_u are 25.72, 11.32, 0.41, 0.94 and 0.92, respectively. Notably, under the assumption a_π=a_y=0, the weights assigned to the remaining five terms are positive. The optimal weights in curve 4 are substantially lower compared to the absolute values of the optimal weights in curve 1.

Curve 5 assumes four free choice variables, and represents the Taylor (1993) rule with scaling, where a_{pi},a_r,a_y,a_π are optimized assuming a_m=a_v=a_u=0. That causes a sum of squares S=0.46065. The corresponding optimal weights a_{pi},a_r,a_π,a_y are 66.14, −10.02, −16.34 and 1.64, respectively. Curve 6 assumes four free choice variables, and represents the Quantity Equation (Friedman, 1970) with scaling, where a_{pi},a_r,a_m,a_v are optimized

Figure 1: The monthly US January 1, 1959-March 31, 2022 empirical interest rate i_t and the interest rate i_t based on (1). Panel a: a_{pi} = 66.72, a_r = –11.44, a_π = –16.84, a_y = –3.13, a_m = 1.29, a_v = 2.73, a_u=2.06. Panel b: The Taylor (1993, 1999) rules. Panel c: The average of the curves 1-7. Panel d: The standard deviation of the curves 1-7. Panel e: The average of curves 1-9. Panel f: The standard deviation of the curves 1-9



assuming $a_\pi=a_y=a_u=0$. That causes a slightly lower sum of squares S=0.45850 compared to curve 5. The corresponding optimal weights a_{pi},a_r,a_m,a_v are 25.82, 10.77, 0.77 and 1.88, respectively. Curve 7 assumes three free choice variables, and represents the Phillips (1958) curve with scaling, where a_{pi},a_r,a_u are optimized assuming $a_\pi=a_y=a_m=a_v=0$. That causes an even lower sum of squares S=0.45780 compared to curves 5 and 6. The corresponding optimal weights a_{pi},a_r,a_u are 25.68, 11.58 and 1.55, respectively. The results show that the Phillips (1958) curve with scaling explains the interest rate i_t better than the Taylor (1993) rule with scaling and the Quantity Equation (Friedman, 1970) with scaling.

Curve 8 represents the Taylor (1993) rule, assuming $a_\pi s_\pi=a_y s_y=0.5$, $a_{pi}s_{pi}=a_r s_r=1$, $a_m=a_v=a_u=0$. That causes a sum of squares S=0.83077. Curve 9 represents the Taylor (1999) rule, assuming $a_\pi s_\pi=a_y s_y=0.5$, $a_{pi}s_{pi}=a_r s_r=1$, $a_m=a_v=a_u=0$. That causes a slightly lower sum of squares S=0.81953. The sum of squares S= 0.44567 in curve 1 is 46.35% and 45.62%, respectively, lower than the Taylor (1993) rule's S=0.83077, and the Taylor (1999) rule's S=0.81953. Hence curve 1 explains the interest rate i_t better than both Taylor (1993, 1999) rules.

"Curve average 1-7" shows the average of curves 1-7. The corresponding sum of squares is S=0.45094, i.e. a 45.72% decrease and a 44.97% decrease, respectively, compared with the Taylor (1993) rule and the Taylor (1999) rule. Finally, "Curve average 1-9" shows the average of curves 1-9. The corresponding sum of squares is S=0.47122, i.e. a 43.27% decrease and a 42.50% decrease, respectively, compared with the Taylor (1993, 1999) rules.

Overall, among the curves 1-7, the weight a_{pi} assigned to the inflation rate π_t the weight a_m assigned to the deviation $Log\left(\frac{m_t}{\bar{m}_t}\right)$ in money supply, the weight a_v assigned to the deviation $Log\left(\frac{v_t}{\bar{v}_t}\right)$ in money velocity, and the weight a_u assigned to the deviation $\bar{u}_t-u_t$ in unemployment rate are always positive. That means that the inflation rate π_t, the deviation $Log\left(\frac{m_t}{\bar{m}_t}\right)$ in money supply, the deviation $Log\left(\frac{v_t}{\bar{v}_t}\right)$ in money velocity, and the deviation $\bar{u}_t-u_t$ in unemployment rate impact the interest rate i_t positively. Notably, the weight a_π assigned to the deviation $\pi_t-\pi_t^*$ in inflation rate is always negative. The weight a_r assigned to the equilibrium real interest rate r_t^*, and the weight a_y assigned to the deviation $Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP are predominantly negative. Hence the equilibrium real interest rate r_t^*, the deviation $Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP may impact the interest rate i_t negatively. These findings differ from the common wisdom, and the Taylor (1993, 1999) rules, that the deviation $\pi_t - \pi_t^*$ in inflation rate and the deviation $Log\left(\frac{y_t}{\bar{y}_t}\right)$ in real GDP impact the interest rate i_t positively.

Figure 1, panel a plots the empirical interest rate i_t with black "+", and curve 9 for the interest rate i_t in (1) with red filled triangles according to Table 1. Panel b plots the Taylor (1993; 1999) rules. Panel c plots the average interest rate i_t of the curves 1-7. Panel d plots the standard deviation of the predicted interest rate i_t of the curves 1-7. Panel e plots the average interest rate i_t of the curves 1-9. Panel f plots the standard deviation of the predicted interest rate i_t of the curves 1-9.

Panel a, curve 1 assumes seven free choice variables, where $a_{pi}, a_r, a_\pi, a_y, a_m, a_v, a_u$ are optimized. It fits the empirical interest rate i_t better than the Taylor (1993, 1999) rules, and has the lowest sum of squares S=0.44567 in Table 1. The local maximum of curve 1 in 1974 is close to the empirical interest rate i_t. Curve 1 shows an especially high interest rate i_t in 1980. In addition, it predicts negative interest rate i_t from April, 2009 to October, 2009. Panel b shows the Taylor (1993; 1999) rules. Overall, the Taylor (1999) rule predicts marginally lower interest rate i_t compared with the Taylor (1993) rule after the maximum in 1980.

Panel c, curve "Average curves 1-7" shows the average interest rate of the curves 1-7. Overall, the predicted interest rate is lower than the empirical interest rate i_t, except after 2010. Furthermore, it predicts negative interest rate i_t from April, 2009 to September, 2009. Panel d, curve "Standard deviation curves 1-7" shows the standard deviation of the interest rate i_t of the curves 1-7. In general, the standard deviation of the 1-7 curves is quite low. It shows moderately high values in 2010, 2020 and 2022.

Panel e, curve "Average curves 1-9" shows the average interest rate i_t of the curves 1-9. Panel f, curve "Standard deviation curves 1-9" shows the standard deviation of the interest rate i_t of the curves 1-9. Overall, the curve "Average curves 1-9" predicts a marginally higher interest rate i_t compared with the "Average curves 1-7". Similarly, the curve "Standard deviation curves 1-9" shows higher interest rate i_t compared to the "Standard deviation curves 1-7". This is because, overall, the Taylor (1993, 1999) rules predict higher interest rate i_t compared with curves 1-7.

4. CONCLUSION

The article establishes a generalized interest rates model by generalizing the Taylor (1993) rule from four terms to seven terms, and scaling the terms relative to each other. First, the article introduces three additional terms, i.e. the deviation in money supply, the deviation in money velocity, and the deviation in unemployment rate, which accounts for the money supply, the money velocity, and the unemployment rate, respectively. Second, the article investigates the seven combinations of the Taylor (1993) rule, the Quantity Equation (Friedman, 1970), and the Phillips (1958) curve, allowing the presence of one rule, two rules, or all three rules. Third, the article innovatively explores the scaling issue within the seven terms, i.e. the inflation rate, the equilibrium real interest rate, the deviation in inflation rate, the deviation in real GDP (Gross Domestic Product), the deviation in money supply, the deviation in money velocity, and the deviation in unemployment rate. To our best knowledge, the article investigates the scaling issue for the first time related to the Taylor (1993) rule's framework. The optimal seven weights are estimated and tested through the monthly January 1, 1959-March 31, 2022 US data. First, the two Taylor (1993, 1999) rules are evaluated against the empirics. Second, the seven combinations of the Taylor (1993) rule, the Quantity Equation (Friedman, 1970), and the Phillips (1958) curve with scaling are explored and tested.

The findings show that, first, all the seven combinations of the Taylor (1993) rule, the Quantity Equation (Friedman, 1970), and the Phillips (1958) curve with scaling give substantially better results than both the Taylor (1993, 1999) rules without scaling. The second best combination is the Taylor (1993) rule and the Phillips (1958) curve with scaling. Third best is the combination of the Taylor (1993) rule and the Quantity Equation (Friedman, 1970) with scaling. Second, when choosing only one rule with scaling, the Phillips (1958) curve is the best, followed by the Quantity Equation (Friedman, 1970), and finally the Taylor (1993, 1999) rules. Third, when choosing between two combinations with scaling, the Taylor (1993) rule and the Phillips (1958) curve is the best, followed by the Taylor (1993) rule and the Quantity Equation (Friedman, 1970), and finally, the Quantity Equation (Friedman, 1970) and the Phillips (1958) curve.

Among the seven terms, the most explanatory term to the interest rate is the inflation rate. The weights assigned to the inflation rate are always positive. Thus, it impacts the interest rate positively. The second explanatory term is the deviation in inflation rate, and the equilibrium real interest rate. Notably, the deviation in the inflation rate impacts the interest rate negatively. The weights assigned to the equilibrium real interest rate are predominantly negative. Thereafter, with decreasing degrees of negativity, followed by the deviation in real GDP, the deviation in money velocity, the deviation in unemployment rate, and the deviation in money supply. Thus, the money velocity is more explanatory for the interest rate than the money supply. The weights assigned to the deviation in real GDP are also predominantly negative. The deviation in money velocity, the deviation in unemployment rate, and the deviation in money supply impact the interest rate positively.

Future research may compare the empirics for different geographical regions, and incorporate the monetary policy changes over different time periods. Further possibilities are to account for the uncertainty and variation of the potential real GDP, the real equilibrium interest rate, and the natural unemployment rate. Alternative methods may be assessed to better estimate these three terms. Future research may also investigate the interest rate by incorporating time series approaches, or intruding broader financial theories.

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APPENDIX A

Nomenclature

Parameters

a_{pi} Weight assigned to the inflation rate, $-\infty \leq a_{pi} \leq \infty$
a_r Weight assigned to the equilibrium real interest rate, $-\infty \leq a_r \leq \infty$
a_π Weight assigned to the deviation in inflation rate, $-\infty \leq a_p \leq \infty$
a_y Weight assigned to the deviation in real GDP, $-\infty \leq a_y \leq \infty$
a_m Weight assigned to the deviation in money supply, $-\infty \leq a_m \leq \infty$
a_v Weight assigned to the deviation in money velocity, $-\infty \leq a_v \leq \infty$
a_u Weight assigned to the deviation in unemployment rate, $-\infty \leq a_u \leq \infty$
s_{pi} Scaling parameter for the inflation rate, $s_{pi} > 0$
s_r Scaling parameter for the equilibrium real interest rate, $s_r > 0$
s_π Scaling parameter for the deviation in inflation rate, $s_\pi > 0$
s_y Scaling parameter for the deviation in real GDP, $s_y > 0$
s_m Scaling parameter for the deviation in money supply, $s_m > 0$
s_v Scaling parameter for the deviation in money velocity, $s_v > 0$
s_u Scaling parameter for the deviation in unemployment rate, $s_u > 0$

Variables

i_t Interest rate at time t, $i_t \in R$
π_t Inflation rate, $\pi_t \in R$
π_t^* Target inflation rate, $\pi_t^* \in R$
r_t^* Equilibrium real interest rate, $r_t^* \in R$
y_t Real GDP (Gross Domestic Product), $y_t \geq 0$
$\overline{y}_t$ Real potential GDP, $\overline{y}_t \geq 0$
m_t Money supply at time t, $m_t > 0$
u_t Unemployment rate, $u_t \geq 0$
$\overline{u}_t$ Natural rate of unemployment, $\overline{u}_t \geq 0$
t Time, $t \geq 0$