



Loss Given Default Estimating by the Conditional Minimum Value

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ABSTRACT

The Basel Committee offers banks the opportunity to estimate loss given default (LGD) if they wish to calculate their own value for the capital required to cover credit losses. The flexibility to determine LGD values tailored to a bank's portfolio will likely be a motivation for a bank to want to move from the foundation to the advanced internal ratings-based approach. The importance of estimating LGD stems from the fact that a lender's expected loss is the product of the probability of default, the credit exposure at the time of default and the LGD. The Mertonian approach is used for LGD estimation. In this paper, we estimated the (LGD) parameter, using the Merton model, by the introduction of a new parameter which called the conditional minimum value. Four components have been developed in this work: Estimation of conditional minimum, estimation of the LGD, development of a practical component, and finally validation of the proposed model.

Keywords: Credit Risk Modeling, Loss Given Default, Rating Model, Basel 2, Merton's Model, Backtesting

JEL Classifications: G17, G21, G24, G28, G32, G38

1. INTRODUCTION

Loss given default (LGD) is a common parameter in risk models and also a parameter used in the calculation of economic capital, expected loss or regulatory capital under Basel II for a banking institution. It's one of the most crucial key parameters needed to evaluate the expected and unexpected credit losses necessary for credit pricing as well as for calculation of the regulatory Basel requirement. While the credit rating and probability of default (PD) techniques have been advancing in recent decades.

A lot of focus has been devoted to the estimation of PD while LGD has received less attention and has at times been treated as a constant. Das and Hanouna (2008) mentioned that using constant loss estimates could be misleading inasmuch as losses vary a great deal. According to Moody's 2005 findings; average recovery rates, defined as $1 - LGD$, can vary between 8% and 74% depending on the year and the bond type. For sophisticated risk management, LGD undoubtedly needs to be assessed in greater detail.

If a bank uses the advanced internal ratings-based (IRB) approach, the Basel II accord allows it to use internal models to estimate the LGD. While initially a standard LGD allocation

may be used for the foundation approach, institutions that have adopted the IRB approach for PD are being encouraged to use the IRB approach for LGD because it gives a more accurate assessment of loss. In many cases, this added precision changes capital requirements.

This paper is formulated into two sections:

The theoretical section, which has highlighted the overall LGD estimation models in recent decades as well as a theoretical model proposed by way of:

- Introducing a new parameter which be called the conditional minimum for an asset, based on the Merton model.
- Elaborating a mathematical development to estimate LGD using the conditional minimum value.
- A detail will be provided in the model developed to specify the LGD formula in the case of a single asset then again in the case of several assets.

The practical Section, which includes:

- An application made according to the proposed model using actual data from a Moroccan bank. This application will be done in two cases: Single asset then again in several assets

to highlight the effect of the correlation of assets that could minimize LGD rates.

- A Backtesting program will be conducted to check the estimated power of the proposed model.
- A comparison of the model developed with the model that uses the minimum value introduced in Ammari and Lakhnati (2016).

2. LITERATURE REVIEW

2.1. LGD Estimation Models

LGD has attracted little attention before the 21st century; one of the first papers on the subject written by Schuermann (2004) provides an overview of what was known about LGD at that time. Since the first Basel II consultative papers were published there has been an increasing amount of research on LGD estimation techniques: Altman et al. (2004); Frye (2003); Gupton (2005); Huang and Oosterlee (2008) (Table 1).

One of the last models produced to estimate the LGD is the LossCalc model introduced by Moody’s KMV. The general idea for estimating the recovery rate is to apply a multivariate linear regression model including certain risk factors, e.g., industry factors, macroeconomic factors, and transformed risk factors resulting from ”mini-models.”

Another estimation model proposed by Steinbauer and Ivanova (2006), consists of two steps, namely a scoring and a calibration step. The scoring step includes the estimation of a score using collateralization, haircuts, and expected exposure at default (EAD) of the loan and recovery rates of the uncollateralized exposure. The score itself can be interpreted as a recovery rate of the total loan but is only used for relative ordering in this case.

2.2. Risk Weighted Assets

Risk-weighted assets are computed by adjusting each asset class for risk in order to determine a bank’s real world exposure to potential losses.

Regulators then use the risk weighted total to calculate how much loss-absorbing capital a bank needs to sustain it through difficult markets. Under the Basel III rules, banks must have top quality capital equivalent to at least 7% of their risk-weighted assets or they could face restrictions on their ability to pay bonuses and dividends.

The risk weighting varies accord to each asset’s inherent potential for default and what the likely losses would be in case of default - so a loan secured by property is less risky and given a lower multiplier than one that is unsecured.

Table 1: The various models for LGD estimation

Model	The estimated LGD function
Frye and Jacobs (2012)	$N\left[N^{-1}(cDR) - k \right] / \sqrt{1-\rho}$ <p>cDR is the conditionally expected default rate, ρ is the correlation assets</p> $k = \text{LGD risk index} = [N^{-1}(PD) - N^{-1}(EL)] / \sqrt{1-\rho}$ <p>N is the normal distribution standard function N⁻¹ is the normal distribution standard function inverse</p>
Frye (2010)	$1 - \left[\mu + \frac{\sigma q \left[\sqrt{1-\rho} N^{-1}(cDR) - N^{-1}(PD) \right]}{\sqrt{\rho}} \right]$ <p>μ = recovery mean, σ = recovery SD, q = recovery sensitivity</p>
Pykhtin (2016)	$N\left[\frac{-\mu - \beta Y}{\sqrt{1-\beta^2}} \right] - \text{Exp}\left[\mu + \sigma \beta Y + \frac{\sigma^2}{2}(1-\beta^2) \right] * N\left[\frac{-\mu - \beta Y}{\sqrt{1-\beta^2}} - \sigma \sqrt{1-\beta^2} \right]$ $Y = [(PD) - \sqrt{1-\rho} N^{-1}(cDR)] / \sqrt{\rho}$ <p>μ = log recovery mean, σ = log recovery SD, β = recovery sensitivity</p>
Giese (2005)	$1 - a_0 (1 - PD^{a_1})^{a_2}$ <p>a₀, a₁, a₂ = value to be determined</p>
Hillebrand (2006)	$\int_{-\infty}^{\infty} N\left(\alpha - \frac{bdc}{e} + \frac{bd}{e} N^{-1}[cDR] - b\sqrt{1-d^2}x \right) N(x) dx$ <p>a, b = parameters of LGD is second factor; d = correlation of factors;</p> $c = \frac{N^{-1}(PD)}{\sqrt{1-\rho}} \quad e = \frac{\sqrt{\rho}}{\sqrt{1-\rho}}$

The formula for calculating RWA is in the form of:

$$RWA = K * EAD \tag{1}$$

EAD¹: Is seen as an estimation of the extent to which a bank may be exposed to counterparty in the event of, and at the time of, that counterparty's default. EAD is equal to the current amount outstanding in case of fixed exposures like term loans. In our calculation the value of LGD was set at 45%.

K: Is the capital requirement is in the form of:

$$K = \left[LGD * \varphi \left[\frac{1}{(1-\rho)^{0.5}} * \varphi^{-1}(PD) + \left(\frac{\rho}{1-\rho} \right)^{0.5} * \varphi^{-1}(0,999) \right] - PD * LGD \right] * 12,5 \tag{2}$$

PD is the probability that the borrower falls default;
LGD is the loss rate in the presence of a default

The PD_i corresponds to :

$$PD_i = P[A_i < D]$$

With :

PD_i is the probability that the borrower i falls default

A_i is the asset value i:

D is the value of obligations i:

φ is the normal distribution standard function;

φ⁻¹ is the normal distribution standard function inverse.

The PD corresponds to:

$$PD_i = P[A_i < B_i] \tag{3}$$

With:

A_i is the asset value i

B_i is the value of obligations i

N is the cumulative normal distribution function

N⁻¹ is the cumulative normal distribution function inverse.

3. MODEL SPECIFICATION

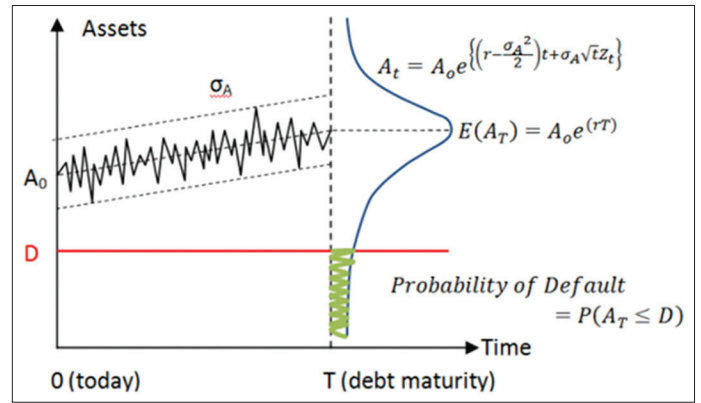
3.1. Estimating the Expected LGD

Merton (1974) and Black and Scholes (1973) proposed a simple model of the firm that provides a way of relating credit risk to the capital structure of the firm. In this model the value of the firm's assets is assumed to follow a lognormal diffusion process with a constant volatility (Chart 1).

$$A_{i,t} = A_{i,0} e^{\left(\left(\mu_i - \frac{\sigma_i^2}{2} \right) t + \sigma_i X_t \right)} \tag{4}$$

¹ Draft Supervisory Guidance on Internal Ratings-Based Systems for Corporate Credit.

Chart 1: Point of default



$$X_t \sim N(0, \sqrt{t}) \tag{5}$$

X_t un processus de Wiener avec une attente de 0 et de variance t.

μ_i represents the expected rate of return on assets

σ_i is the volatility of the return on assets.

$$\ln(A_{i,t}) = \ln(A_{i,0}) + \left(\left(\mu_i - \frac{\sigma_i^2}{2} \right) t + \sigma_i X_t \right) \tag{6}$$

$$\Rightarrow \ln(A_{i,t}) \sim N \left(\ln(A_{i,0}) + \left(\mu_i - \frac{\sigma_i^2}{2} \right) t, \sigma_i \sqrt{t} \right) \tag{7}$$

So A_{i,t} follows a lognormal distribution with parameters

$\ln(A_{i,0}) + \left(\mu_i - \frac{\sigma_i^2}{2} \right) t$ and $\sigma_i \sqrt{t}$ with a density function

$$g(x) = \frac{1}{x} \frac{1}{b\sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{\ln(x)-a}{b} \right)^2}$$

$$a = \ln(A_{i,0}) + \left(\mu_i - \frac{\sigma_i^2}{2} \right) t \text{ et } b = \sigma_i \sqrt{t}$$

It is possible to calculate expectancy of A_{i,t} according to the log normal distribution.

$$\mu_{A_{i,t}} = E(A_{i,t}) = e^{\ln(A_{i,0}) + \left(\mu_i - \frac{\sigma_i^2}{2} \right) t + \sigma_i^2 \cdot \frac{t}{2}} \tag{8}$$

$$\mu_{A_{i,t}} = A_{i,0} \cdot e^{\mu_i t} \tag{9}$$

3.2. Conditional Minimum Value

Ammari and Lakhnati (2016) has introduced the notion of the minimum value of an asset to estimates the LGD:

For a fixed probability α, Min_α is defined by

$$P(A_{i,t} < \text{Min}_\alpha) = \alpha \tag{10}$$

Now we introduce a new notion $CMin_{A_i, \alpha}$ which be called conditional minimum value:

$$E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) = CMin_{A_i, \alpha} \tag{11}$$

Which simply represents the minimum conditional value that an asset may have during a period, with a risk α

In the case of Merton model we have:

$$E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) = \int_0^{Min_{A_{i,T}, \alpha}} A_{i,t} g(A_{i,t}) dA_{i,t} / P(A_{i,t} < Min_{A_{i,T}, \alpha}) \tag{12}$$

$$= \frac{1}{\alpha} \int_0^{Min_{A_{i,T}, \alpha}} A_{i,t} \frac{1}{A_{i,t}} \cdot \frac{1}{b\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\ln(A_{i,t})-a}{b})^2} dA_{i,t} \tag{13}$$

$$= \frac{1}{\alpha} \int_0^{Min_{A_{i,T}, \alpha}} \frac{1}{b\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{\ln(A_{i,t})-a}{b})^2} dA_{i,t} \tag{14}$$

$$= \frac{1}{\alpha} \int_{-\infty}^{\ln(Min_{A_{i,T}, \alpha})} \frac{e^{A_{i,t}}}{b\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{A_{i,t}-a}{b})^2} dA_{i,t} \tag{15}$$

$$= \frac{1}{\alpha} \int_{-\infty}^{\ln(Min_{A_{i,T}, \alpha})} \frac{e^{A_{i,t}-\frac{1}{2}(\frac{A_{i,t}-a}{b})^2}}{b\sqrt{2\pi}} dA_{i,t} \tag{16}$$

We have

$$\begin{aligned} A_{(i,t)} - \frac{1}{2} \left(\frac{A_{i,t}-a}{b} \right)^2 &= -\frac{1}{2b^2} (A_{i,t}^2 - 2aA_{i,t} + a^2 - 2b^2A_{i,t}) \\ &= -\frac{1}{2b^2} (A_{i,t}^2 - 2A_{i,t}(a+b^2) + a^2) \\ &= -\frac{1}{2b^2} (A_{i,t} - (a+b^2))^2 + -\frac{1}{2b^2} (a^2 - (a+b^2)^2) \\ &= -\frac{1}{2b^2} (A_{i,t} - (a+b^2))^2 + a + \frac{b^2}{2} \end{aligned} \tag{17}$$

$$\begin{aligned} E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) &= \frac{1}{\alpha} \int_{-\infty}^{\ln(Min_{A_{i,T}, \alpha})} \frac{e^{-\frac{1}{2b^2}(A_{i,t} - (a+b^2))^2 + a + \frac{b^2}{2}}}{b\sqrt{2\pi}} dA_{i,t} \\ &= \frac{e^{a+\frac{b^2}{2}}}{\alpha} \int_{-\infty}^{\ln(Min_{A_{i,T}, \alpha})} \frac{e^{-\frac{1}{2}(\frac{A_{i,t} - (a+b^2)}{b})^2}}{b\sqrt{2\pi}} dA_{i,t} \end{aligned} \tag{18}$$

We set $s = \left(\frac{A_{i,t} - (a+b^2)}{b} \right)$
 $\Rightarrow ds = \frac{dA_{i,t}}{b}$

So

$$\begin{aligned} E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) &= \frac{e^{a+\frac{b^2}{2}}}{\alpha} \int_{-\infty}^{\left(\frac{\ln(Min_{A_{i,T}, \alpha}) - (a+b^2)}{b}\right)} \frac{e^{-\frac{1}{2}s^2}}{\sqrt{2\pi}} ds \\ &= \frac{e^{a+\frac{b^2}{2}}}{\alpha} \Phi\left(\frac{\ln(Min_{A_{i,T}, \alpha}) - (a+b^2)}{b}\right) \end{aligned} \tag{19}$$

Φ is the normal distribution standard function;

Ammari and Lakhnati (2016) proved that $Min_{A_{i,T}, \alpha} = a + b\Phi^{-1}(\alpha)$

With Φ^{-1} is the normal distribution standard function inverse.

So

$$E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) = \frac{e^{a+\frac{b^2}{2}}}{\alpha} \Phi\left(\frac{a + b\Phi^{-1}(\alpha) - (a+b^2)}{b}\right)$$

From $E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) = \frac{e^{a+\frac{b^2}{2}}}{\alpha} \Phi(\Phi^{-1}(\alpha) - b)$ (20)

We have $a = \ln(A_{i,0}) + \left(\mu i - \frac{\sigma i^2}{2} \right) \cdot t$ and $b = \sigma i \sqrt{t}$

We deduce that

$$E(A_{i,t} | A_{i,t} < Min_{A_{i,T}, \alpha}) = \frac{e^{\ln(A_{i,0}) + \mu i \cdot t}}{\alpha} \Phi\left(\Phi^{-1}(\alpha) - \sigma i \sqrt{t}\right) \tag{21}$$

at $t = T=1$

$$CMin_{A_{i,T}, \alpha} = \frac{e^{\ln(A_{i,0}) + \mu i}}{\alpha} \Phi\left(\Phi^{-1}(\alpha) - \sigma i\right) \tag{22}$$

The formula (22) is very useful for financial calculations under the conditionnel minimum value that could reach the asset A_i at any time t , specifically at the maturity T , which can be regarded as an expected conditionnal value according to a previously specified risk level.

3.3. Estimated Loss Rate (LGD)

LGD is calculated in various ways, but the most popular is Gross LGD, where total losses are divided by EAD. An alternate method is to divide losses by the unsecured portion of a credit line (where security covers a portion of EAD. This is known as Blanco LGD. If the collateral value is zero in the last case then Blanco LGD is equivalent to gross LGD. A variety of statistical methods may be applied.

In this article, the rate of LGD will be calculated according to the minimum value.

With the formula (22), we can already get an idea of the impairment of financial assets over time (t), which is essential to calculate the rate of percentage loss of the initial value of a financial asset.

In this section, a development of the formula (22) will be established by calculating loss rates (LGD) that could represent a financial asset.

The Chart 2 revealed two losses of asset $A_{i,t}$, an average loss and other unexpected with a level of risk α .

With α lower level of risk, it is possible to calculate an unexpected loss as in the previous section. This loss will be used to determine the unexpected loss rate with the use of the initial value of the asset A as:

Estimation de la LGD:

$$LGD_{A_{i,t},\alpha} = \frac{A_{i,0} - CMin_{A_{i,T},\alpha}}{A_{i,0}} \tag{23}$$

$$LGD_{A_{i,t},\alpha} = \frac{A_{i,0} - \left(\frac{e^{\ln(A_{i,0}) + \mu_i}}{\alpha} \right) \varphi(\varphi^{-1}(\alpha) - \sigma_i)}{A_{i,0}} \tag{24}$$

$$LGD_{A_{i,t},\alpha} = 1 - \frac{e^{\mu_i} \varphi(\varphi^{-1}(\alpha) - \sigma_i)}{\alpha} \tag{25}$$

3.3.1. Case of a single asset A_i

When $t = T$

$$LGD_{\alpha} = 1 - \frac{e^{\mu_i} \varphi(\varphi^{-1}(\alpha) - \sigma_i)}{\alpha} \tag{26}$$

α is the risk taken on assets.

3.3.2. Case of two assets

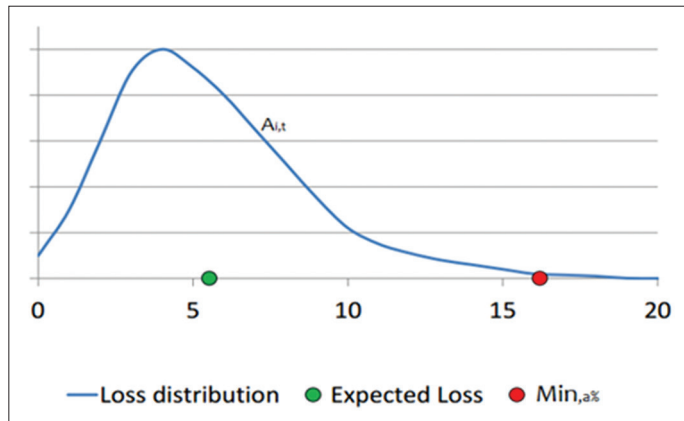
$$CMin_{A_{i,t} + A_{j,t}, \alpha} = \frac{e^{\ln(A_{i,0} + A_{j,0}) + (w_i \mu_i + w_j \mu_j)}}{\alpha} \varphi(\varphi^{-1}(\alpha) - \sigma_{ij}) \tag{27}$$

w_i, w_j are weights of the assets i, j

$$\sigma_{ij} = \sqrt{w_i^2 \sigma_i^2 + 2 * \rho * w_i * w_j \sigma_i \sigma_j + w_j^2 \sigma_j^2}$$

ρ is the correlation between $A_{i,t}$ and $A_{j,t}$

Chart 2: Unexpected loss



$$LGD_{A_i + A_j, \alpha} = 1 - \frac{e^{w_i \mu_i + w_j \mu_j} \varphi(\varphi^{-1}(\alpha) - \sigma_{ij})}{\alpha} \tag{28}$$

3.3.3. Case of several credit portfolio as well

$$LGD_{\sum_{i=1}^p A_i, \alpha} = 1 - \frac{e^{\sum_{i=1}^p w_i \mu_i} \varphi(\varphi^{-1}(\alpha) - R)}{\alpha} \tag{29}$$

Such as $R = t_w \Sigma w$

Σ is the variance covariance matrix of the assets

w is the weights matrix of the assets

t_w is the transposed weights matrix of the assets.

4. EMPIRICAL RESULTS

4.1. Illustration of the Calculation of the Conditional Minimum Value and the LGD

4.1.1. Case of a single asset

$$\text{With } \sigma_i^2 = \frac{\sum_{j=1}^n (\mu_{i,j} - \bar{\mu}_i)^2}{n-1} \text{ and } \bar{\mu}_i = \frac{\sum_{j=1}^n \mu_{i,j}}{n}$$

$\sigma_i = 16.35\%$ and $\bar{\mu}_i = 7.42\%$

We would calculate $CMin_{A_{i,T},\alpha}$ with $\alpha = 1\%$ as a risk level from the 5th year, posing $A_{i,0} = 7.000.000$ Dhs (Table 2).

The Chart 3 shows the distribution of asset A_i , with $T = 1.800$ (for 3 years) according to a number of simulations, the final value of $CMin_{A_{i,T},1\%} = 4.817.437$ DH with $LGD 1\% = 31.18\%$ which is equivalent to the A_i loss percentage.

4.1.2. LGD of two assets A_i and A_j

The financial data of a portfolio of two companies are shown in Table 3.

In this case, we have:

$$A_{i,0} = 7.000.000 \mu_i = 7.42\% \text{ and } \sigma_i = 16.35\% w_i = 0.38$$

Table 2: Financial data of a Moroccan bank

Company	Year	Tutnover (MAD)	Assets (MAD)	Rate of return (%)
C1	1	17,500,000	7,000,000	
	2	16,250,000	6,500,000	-7
	3	20,000,000	8,000,000	23
	4	18,750,000	7,500,000	-6
	5	22,500,000	9,000,000	20
Average return	7.42%			
Volatility	16.35%			

Table 3: The financial data of a portfolio of two companies

Company	Year	Turnover (MAD)	Assets (MAD)	Rate of return (%)	Average return (%)	Volatility (%)	Asset correlation (%)
C1	1	17,500,000	7,000,000		7.42	16.35	-59
	2	16,250,000	6,500,000	-7			
	3	20,000,000	8,000,000	23			
	4	18,750,000	7,500,000	-6			
	5	22,500,000	9,000,000	20			
C2	1	22,500,000	9,000,000		15.84	26.94	
	2	23,750,000	9,500,000	6			
	3	21,250,000	8,500,000	-11			
	4	32,500,000	13,000,000	53			
	5	37,500,000	15,000,000	15			

Chart 3: Determination of the minimum value of the asset

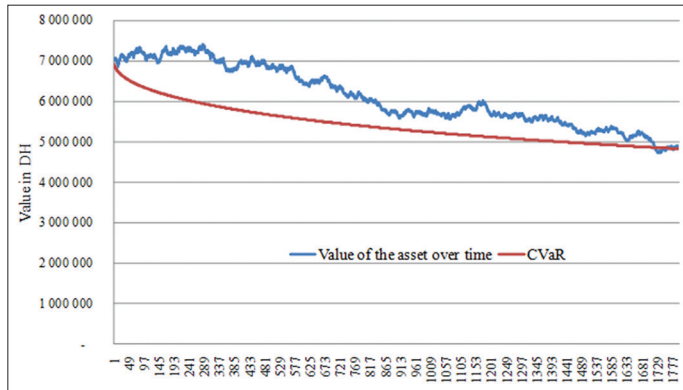


Chart 4: Estimated loss rate LGD

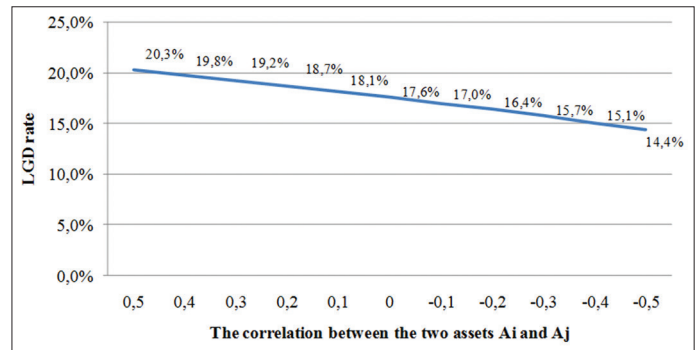
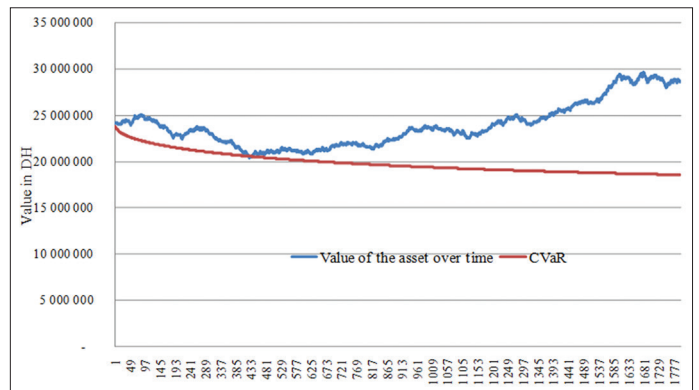


Chart 5: Back testing of the calculated conditional minimum value, $\alpha = 5\%$



$$A_{j,0} = 15.000.000 \quad \mu_i = 15.84\% \text{ and } \sigma_i = 26.94\% \quad w_j = 0.62$$

Case 1: Separated calculation $LGD_{A_{i,T}}$ and $LGD_{A_{j,T}}$

$$CMin_{A_{i,T}, 1\%} = 4.817.437 \text{ DH} \quad CMin_{A_{j,T}, 1\%} = 8.293.882 \text{ DH}$$

$$LGD_{A_{i,T}, 1\%} = 31.18\% \quad LGD_{A_{j,T}, 1\%} = 44.71\%$$

$$Min_{A_{i,T}, 1\%} + Min_{A_{j,T}, 1\%} = 13.111.319 \text{ DH}$$

$$LGD_{A_{i,T} \text{ et } A_{j,T}, 1\%}$$

$$\left(\text{separated calculation of } LGD_{A_{i,T}} \text{ and } LGD_{A_{j,T}} \right) = 40.40\%$$

Case 2: Calculation of LGD as in a portfolio of credit assets

So we use the formula below,

$$CMin_{A_{i,t}+A_{j,t}, \alpha} = \frac{e^{\ln(A_{i,0} + A_{j,0}) + (w_i \cdot \mu_i + w_j \cdot \mu_j)}}{\alpha} \cdot \Phi\left(\Phi^{-1}(\alpha) - \sigma_{ij}\right)$$

w_i, w_j are weights of the assets i, j

$$\sigma_{ij} = \sqrt{w_i^2 \cdot \sigma_i^2 + 2 \cdot \rho \cdot w_i \cdot w_j \cdot \sigma_i \cdot \sigma_j + w_j^2 \cdot \sigma_j^2}$$

$$\text{Asset correlation } \rho = -59\%$$

So:

$$\sigma_{ij} = 13.72\%$$

$$\text{And } CMin_{A_{i,T} + A_{j,T}, 1\%} = 18.514.342 \text{ Dhs,}$$

$$LGD_{A_{i,T} + A_{j,T}, 1\%} = 22.86\%$$

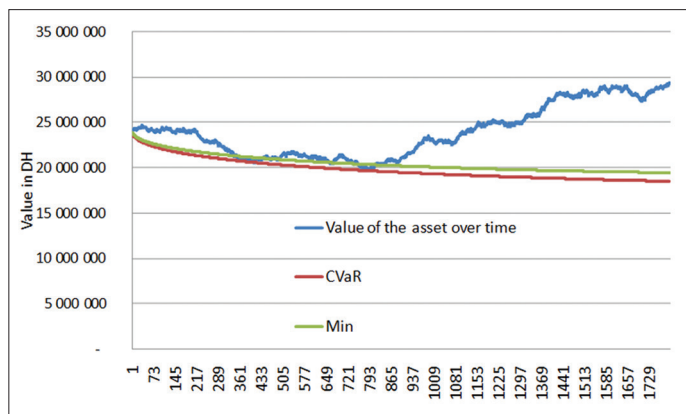
With the second case, a great gain was obtained by the application of the notion of correlation, this is also by the reduction of the estimated loss rate, which is decreased from 44.44% to 22.86%.

The Chart 4 shows the variation in the estimated LGD loss rate due to the correlation between the two assets A_i and A_j .

It should be noted that the link between the correlation between the two assets and the LDG loss rate is perfectly decreasing, a

Table 4: Back testing based on number of simulations

Element	Number of simulation					
	100		1000		10,000	
Confidence level (%)	5	1	5	1	5	1
Overrun percentage (%)	6.10	1.25	5.20	1.10	5	1
Quality of significance (%)	78	75	96	90	100	100
	77		93		100	

Chart 6: Comparison between Min and CMin

10% decrease in the correlation implies an average reduction of 6% for the loss rate.

4.2. Back testing of the calculated Conditional Minimum Value

Chart 5 and Table 4 show two simulations of the assets distribution in two Ai risk levels 1% and 5%, T = 1000.

The objective of this section is to develop a backtesting program for the developed model. It is shown that the greater the number of simulations the greater the importance of estimated power.

For 100 simulations, the exceedance rate is 6.20% for a level of risk of 5%, which is a quality of 76% significance.

For 10.000 simulations, the model becomes more significant with a quality of 99.10%, the exceedance is 5.04% for a risk of 5% and 0.99% for a 1% risk.

4.3. Comparison Between Min and CMin

We compared the relevance of our model with the model that uses the minimum value to estimate a LGD. The result of this comparison is illustrated in the Chart 6.

By observing the graph above, it should be noted that the CMin model gives more precision of the potential loss of the asset.

5. CONCLUSIONS

In this article, we estimated the LGD rate, using the Merton model, by introducing a new parameter that called the conditional minimum value.

Four components were developed in this work: Estimation of the conditional minimum value that an asset could have during a period, estimation of LGD loss rate, and finally validation of the proposed model.

We can say that the proposed model makes it possible to calculate the LGD rate in a very simple way, in addition, this model takes into account the notion of diversification of portfolios, which was shown by the result section.

It has also been shown that this model gives more accuracy on the LGD rate than the model based on the Min value. It should be noted that this is possible to develop a model based on a stressed minimum value, as a research axis.

Another factor requires a particular analysis, namely the correlation of the assets, this element must be modeled in an adequate way to calculate the weighted assets, in this sense we can cite the work of AMMARI and Lakhnati (2017).

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