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How Useful are the Various Volatility Estimators for Improving GARCH-based Volatility Forecasts? Evidence from the Nasdaq-100 Stock Index

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ABSTRACT: Given the rapid growth of financial markets over the past 20 years, along with the explosive development of financial derivatives, an ever-growing need for accurate and efficient volatility forecasting has emerged. Such forecasts have numerous financial applications, such as value-at-risk, hedge ratio, option price and portfolio selection. Recently, the broad availability of intraday trading data has inspired practitioners to investigate their information content in modeling and forecasting the volatility of financial markets. This study aims to propose the introduction of various volatility estimators (overnight volatility (Brooks et al., 2000), PK (Parkinson, 1980), GK (Garman and Klass, 1980), RS (Rogers and Satchell, 1991), RV (Andersen and Bollerslev, 1998), RBP (Barndorff-Nielsen and Shephard, 2004), and VIX) into the conditional variance of GARCH(1,1) model to explore the information value of those estimators for improving out-of-sample volatility forecasts of Nasdaq-100 stock index returns at daily horizon over the period from 2005 to 2013. Empirical results indicate that the inclusion of each volatility estimator considered in this research shows an improvement in the GARCH model with certain degree, except for the overnight volatility (ONV) estimator. In addition, daily ranges (PK, GK, RS) and realized volatilities (RV, RBP) are far more informative than the volatility index (VIX).

Keywords: GARCH; Information value; Volatility estimator; VIX
JEL Classifications: C52; C53; G32

1. Introduction

Given the rapid growth of financial markets over the last two decades, along with the explosive development of financial derivatives, an ever-growing need for accurate and efficient volatility forecasting has emerged. Such forecasts have numerous financial applications, such as value-at-risk, hedge ratio, option price and portfolio selection. Recently, various financial disasters, such as the collapse of Baring's Bank (1995), the near bankruptcy of Long Term Capital Management (1998), the recent bailouts of Bear Sterns, Lehman Brothers and American International Group (2007), have further highlighted the significance of volatility forecasting, and particularly its crucial role in risk management aspect.

Researchers have long been aware that returns volatility changes over time and that period of high volatility tend to be found in clusters. The autoregressive conditional heteroskedastic (ARCH) model of Engle (1982) and the subsequent generalized autoregressive conditional heteroskedastic (GARCH) model proposed by Bollerslev (1986) respond to address these stylized phenomena.

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Brooks et al. (2000) propose the overnight volatility (ONV) to capture accumulated overnight information which would be useful for capturing the persistence in the conditional heteroscedasticity of stock returns. Motivated by the daily price range, Parkinson (1980) uses the scaled high-low price ranges to develop the daily PK volatility estimator based on the assumption that intraday prices follow a Brownian motion process. Garman and Klass (1980) develop the GK estimator by using opening and closing prices in addition to price range, with assumptions similar to those of the PK estimator. Furthermore, Rogers and Satchell (1991) propose an estimator, RS, by including the drift in the price process. The RS estimator is claimed to be more efficient than PK and GK when stock price has a drift. Andersen and Bollerslev (1998) develop the realized volatility (RV) which is based on the cumulative squared returns from intraday data can offer a good proxy for the latent daily return volatility. Barndorff-Nielsen and Shephard (2004) introduce the realized bipower variation (RBP) estimator which is more efficient than the RV in the presence of jumps. Other possible explanatory variable for volatility is the implied volatility index (VIX) which is derived from option price data with an option pricing model (Koopman et al., 2005; Corrado and Truong, 2007; Khan et al., 2013).

Recently, the broad availability of intraday trading data has inspired practitioners to investigate their information content in modeling and forecasting the volatility of financial markets (Blair et al., 2001; Koopman et al., 2005; Corrado and Truong, 2007; Vipul and Jacob, 2007; Fuertes et al., 2009). Thus, this study aims to propose the introduction of various volatility estimators mentioned above (ONV, PK, GK, RS, RV, RBP, and VIX) into the conditional variance of the GARCH(1,1) model to explore the information value of those estimators for improving out-of-sample volatility forecasts of Nasdaq-100 stock index returns at daily horizon over the period from 2005 to 2013.

The remainder of this study is organized as follows. The empirical model is provided in Section 2, followed in Section 3 by a description of the data employed and the empirical results of the volatility forecasting. Conclusions drawn from this study are summarized in the final Section.

2. Econometric Model

We propose the augmented GARCH model which extends the GARCH(1,1) model by including various volatility estimators, respectively, for its variance equation as follows:

$$R_t = \mu + \varepsilon_t\,,\quad \varepsilon_t = \sigma_t z_t\,,\quad z_t\,|_{\Omega_{t-1}} \sim NID(0,1) \tag{1}$$

$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 + \delta v_{t-1} \tag{2}$$

where R_t is daily return; μ denotes the conditional mean of returns; ε_t is the innovation process; z_t is the standardized residual with zero mean and unit variance; σ_t^2 is the conditional variance; v_{t-1} is a volatility estimator made at day $t-1$, including ONV, PK, GK, RS, RV, RBP and VIX. Table 1 provides a synopsis of these volatility estimators.

The volatility forecasting accuracy of competing models over daily horizons are evaluated by using MAE, MSE, RMSE and LL:

$$MAE = \frac{1}{T}\sum_{t=1}^{T} |\,\sigma_t^2 - \hat{\sigma}_t^2\,| \tag{9}$$

$$MSE = \frac{1}{T}\sum_{t=1}^{T} (\sigma_t^2 - \hat{\sigma}_t^2)^2 \tag{10}$$

$$RMSE = \sqrt{\frac{1}{T}\sum_{t=1}^{T} (\sigma_t^2 - \hat{\sigma}_t^2)^2} \tag{11}$$

$$LL = \frac{1}{T}\sum_{t=1}^{T} [\ln(\sigma_t^2) - \ln(\hat{\sigma}_t^2)]^2 \tag{12}$$

where T is the number of forecast data points; σ_t^2 denotes the volatility forecast on day t; $\hat{\sigma}_t^2$ denotes the true daily volatility and can be proxied by the RV or PK.

Table 1. The synopsis of various volatility estimators
This table presents the various volatility estimators employed in this study. O_t, H_t, L_t and C_t denote the opening, high, low, and closing prices at day t, respectively. $R_{t,d}$ denotes the intraday return at time d in a 5-minute interval observed during day t.

Studies	Volatility estimators	
Brooks et al. (2000)	$\hat{\sigma}^2_{ONV,t} = (\ln(O_t / C_{t-1}))^2$	(3)
Parkinson (1980)	$\hat{\sigma}^2_{PK,t} = (4\ln 2)^{-1} \cdot (\ln(H_t / L_t))^2$	(4)
Garman and Klass (1980)	$\hat{\sigma}^2_{GK,t} = 0.511\left(\ln\frac{H_t}{L_t}\right)^2 - 0.019\left[\left(\ln\frac{C_t}{O_t}\right)\left(\ln\frac{H_t \cdot L_t}{O_t^2}\right) - 2\left(\ln\frac{H_t}{O_t}\right)\left(\ln\frac{L_t}{O_t}\right)\right] - 0.383\left(\ln\frac{C_t}{O_t}\right)^2$	(5)
Rogers and Satchell (1991)	$\hat{\sigma}^2_{RS,t} = \left(\ln\frac{H_t}{C_t}\right)\left(\ln\frac{H_t}{O_t}\right) + \left(\ln\frac{L_t}{C_t}\right)\left(\ln\frac{L_t}{O_t}\right)$	(6)
Andersen and Bollerslev (1998)	$\hat{\sigma}^2_{RV,t} = \sum_{d=1}^{D} R^2_{t,d}$	(7)
Barndorff-Nielsen and Shephard (2004)	$\hat{\sigma}^2_{RBP,t} = \frac{\pi}{2} \cdot \sum_{d=2}^{D} \lvert R_{t,d} \rvert \cdot \lvert R_{t,d-1} \rvert$	(8)

3. Data and Empirical Results

3.1 Data description and preliminary analysis

The data examined in this study comprises of daily and intraday data on Nasdaq-100 stock index prices obtained from the Trade and Quote (TAQ) database as well as the VIX data retrieved from the Yahoo Finance website. The sample period for daily data including open, high, low and closing prices spans from 18 August 2005 to 31 July 2013 for a total of 2,001 trading days. The first 1,500 observations (19 August 2005 to 3 August 2011) are used as the in-sample for estimation, while the remaining 500 observations (4 August 2011 to 31 July 2013) are taken as the out-of-sample for forecast evaluation. In addition, the 5-minute intraday trading prices for Nasdaq-100 stock index are used to calculate each of the daily RV and RBP.

Table 2 shows the descriptive statistics of the daily returns for the Nasdaq-100 stock index. The average daily return is negative, and approaches close to zero. The returns series exhibits significant evidence of skewness and kurtosis. That is, the series is skewed to the left, and the distribution of the daily returns is more fat-tailed and high-peaked than normal distribution. The JB statistic further confirms that the daily returns are non-normal distributed. Finally, the Ljung–Box test statistic displays linear dependence for the squared returns and strong ARCH effects.

Table 2. Descriptive statistics of daily returns for the Nasdaq-100 stock index
This table presents the descriptive statistics of daily returns for the Nasdaq-100 stock index. JB represents the statistics of Jarque and Bera (1987)'s normal distribution test. $Q_s(12)$ refers to the Ljung–Box Q statistic of the squared return series for up to the 12th order serial correlation. ** and *** indicate significance at the 5% and 1% levels, respectively.

Mean (%)	Std.	Min	Max	Skew	Kurt	JB	$Q_s(12)$
0.034	1.442	-9.274	11.344	-0.111**	6.635***	3673.149***	1721.530***

3.2 Empirical analysis

Tables 3 ~ 6 present out-of-sample daily volatility forecasts performance across the various models by reporting MAE, MSE, RMSE, LL and benefit statistics, according to both RV and PK volatility proxy measures.

When the volatility proxy is given by RV, both MSE and RMSE statistics of Table 3 indicate that the GARCH-PK model provides the most accurate forecasts, respectively followed from second to seventh place by the GARCH-GK, GARCH-RV, GARCH-RS, GARCH-RBP, GARCH-VIX and GARCH models. The worst-performing model, as measured by the maximum forecast error, is the GARCH-ONV model. MAE also selects the GARCH-PK as the best model for forecasting daily Nasdaq-100 index returns volatility, followed by the GARCH-RV, GARCH-GK, GARCH-RBP, GARCH-RS, GARCH-VIX, GARCH and GARCH-ONV models. As for the forecasting results obtained from the LL statistic, the ranking remains constant with the exception that LL selects GARCH-GK as the second best model and ranks GARCH-RV as the third best one.

Table 3. Out-of-sample volatility forecasts performance
This table presents the volatility forecasting performance when the volatility proxy is given by the RV (Andersen and Bollerslev, 1998), and uses the MAE, MSE, RMSE and LL as evaluation criteria.

Model	MAE	Rank		MSE	Rank		RMSE	Rank		LL	Rank
GARCH	1.002	7		2.490	7		1.578	7		1.164	7
GARCH-ONV	1.009	8		2.521	8		1.587	8		1.170	8
GARCH-VIX	0.969	6		2.403	6		1.550	6		1.093	6
GARCH-PK	0.622	1		1.243	1		1.115	1		0.684	1
GARCH-GK	0.641	3		1.316	2		1.147	2		0.700	2
GARCH-RS	0.681	5		1.521	4		1.233	4		0.743	5
GARCH-RV	0.639	2		1.428	3		1.195	3		0.703	3
GARCH-RBP	0.661	4		1.638	5		1.280	5		0.713	4

Table 4 presents the benefit statistics which measures the information value of various volatility estimators in GARCH-based volatility forecasts when the volatility proxy is given by the RV. Benefit refers to the percentage forecast error reduction that a forecasting model brings relative to the benchmark (GARCH) model according to a given loss function. Take $\text{Benefit}_{\text{MAE}}$ for example, the Benefit criterion of table 4 strongly favors the GARCH-PK model, since its performance is 37.86% more accurate than the GARCH model, while the GARCH-VIX performs marginally better than the benchmark model by about 3.26%.

Table 4. The information value of various volatility estimators
This table shows the information value of various volatility estimators in GARCH-based volatility forecasts when the volatility proxy is given by the RV (Andersen and Bollerslev, 1998), and uses the MAE, MSE, RMSE and LL as evaluation criteria. Benefit_i refers to the percentage forecast error reduction that a forecasting model brings relative to the GARCH model according to loss function i.

Model		$\text{Benefit}_{\text{MAE}}$		$\text{Benefit}_{\text{MSE}}$		$\text{Benefit}_{\text{RMSE}}$		$\text{Benefit}_{\text{LL}}$
GARCH-VIX		3.26%		3.48%		1.76%		6.11%
GARCH-PK		37.86%		50.06%		29.33%		41.21%
GARCH-GK		35.96%		47.13%		27.29%		39.82%
GARCH-RS		31.95%		38.90%		21.83%		36.12%
GARCH-RV		36.20%		42.66%		24.27%		39.62%
GARCH-RBP		34.03%		34.19%		18.87%		38.73%

The evidence suggests that the inclusion of each volatility estimator considered in this study shows an improvement in the GARCH model with certain degree, except for the overnight volatility estimator. In addition, daily ranges (PK, GK, RS) and realized volatilities (RV, RBP) are far more informative than the volatility index. The results remain robust to the PK volatility proxy measure, as showed in Tables 5 ~ 6.

Table 5. Out-of-sample volatility forecasts performance
This table presents the volatility forecasting performance when the volatility proxy is given by the PK (Parkinson, 1980), and also uses the MAE, MSE, RMSE and LL as evaluation criteria.

Model	MAE	Rank		MSE	Rank		RMSE	Rank		LL	Rank
GARCH	1.028	7		2.385	7		1.544	7		1.711	7
GARCH-ONV	1.035	8		2.418	8		1.555	8		1.717	8
GARCH-VIX	0.999	6		2.303	6		1.517	6		1.627	6
GARCH-PK	0.679	1		1.173	1		1.083	1		1.142	1
GARCH-GK	0.699	2		1.252	2		1.119	2		1.164	3
GARCH-RS	0.738	5		1.449	4		1.204	4		1.219	5
GARCH-RV	0.707	3		1.384	3		1.176	3		1.161	2
GARCH-RBP	0.726	4		1.573	5		1.254	5		1.171	4

Table 6. The information value of various volatility estimators
This table shows the information value of various volatility estimators in GARCH-based volatility forecasts when the volatility proxy is given by the PK (Parkinson, 1980), and uses the MAE, MSE, RMSE and LL as evaluation criteria. Benefit$_i$ refers to the percentage forecast error reduction that a forecasting model brings relative to the GARCH model according to loss function *i*.

Model		Benefit$_{MAE}$		Benefit$_{MSE}$		Benefit$_{RMSE}$		Benefit$_{LL}$
GARCH-VIX		2.74%		3.44%		1.74%		4.90%
GARCH-PK		33.87%		50.82%		29.87%		33.26%
GARCH-GK		31.92%		47.51%		27.55%		31.96%
GARCH-RS		28.14%		39.23%		22.05%		28.75%
GARCH-RV		31.23%		41.97%		23.82%		32.14%
GARCH-RBP		29.30%		34.03%		18.78%		31.52%

4. Conclusions

We propose the introduction of various volatility estimators (ONV (Brooks et al., 2000), PK (Parkinson, 1980), GK (Garman and Klass, 1980), RS (Rogers and Satchell, 1991), RV (Andersen and Bollerslev, 1998), RBP (Barndorff-Nielsen and Shephard, 2004), and VIX) into the conditional variance of GARCH(1,1) model to explore the information value of those estimators for improving out-of-sample volatility forecasts of Nasdaq-100 stock index returns at daily horizon over the period from 2005 to 2013. Empirical results indicate that the inclusion of each volatility estimator considered in this research shows an improvement in the GARCH model with certain degree, except for the overnight volatility (ONV) estimator. In addition, daily ranges (PK, GK, RS) and realized volatilities (RV, RBP) are far more informative than the volatility index (VIX). The empirical results presented here are crucial for market practitioners in improving volatility forecasts of financial assets with GARCH models.

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